Article

Profile, Performance and Language in Engineering Mathematics

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Abstract

There is a global concern for retention and success of students in higher education engineering programmes, in particular for students from under-represented communities. Low success in engineering programmes can be partly attributed to students failing mathematics or being unable to articulate mathematics in other engineering courses. This research explores how understanding the academic preparedness of engineering students in relation to their performance in university mathematics can direct curriculum changes to improve student success, driven by the research question: "How can the analysis of student data contribute to understanding student performance in calculus?" Data from engineering students in an extended curriculum programme at the University of Cape Town (UCT) were analysed to generate profiles from variables including gender, home language and performance in university admissions tests. Profiles were related to performance in three consecutive engineering mathematics courses. To determine which variables had the greatest explanatory power on engineering mathematics scores, relative importance analysis was applied. There was no evidence that weaknesses in terms of pre-university mathematics performance held students back from succeeding in the first two engineering mathematics courses at UCT, at least within the support context of the extended curriculum programme. When analysing according to engineering mathematics performance levels (e.g., fail versus first-class pass), academic literacy and, to a lesser extent, quantitative literacy emerged as having greater relative importance than pre-university mathematics in explaining the

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variance in engineering mathematics scores. The findings imply that interventions to improve the success of engineering students should include developing academic literacy practices, potentially in first- and second-year mathematics courses. We reflect on how the relative importance analysis of student data strengthens similar findings from other research on the importance of language in mathematics by highlighting the most important variables explaining students' mathematics performance.

Keywords: curriculum change; profile; mathematics; vector calculus; academic literacy; diagnostic assessment; engineering students; relative importance analysis; student success; language in mathematics

Introduction

In line with economic and political incentives to diversify the engineering workforce, the goals of organisations such as the Council for the Built Environment in South Africa (2019) include increasing the number of engineers from under-represented population groups. The benefits of greater diversity in engineering classes include improved efficiency and effectiveness in problem-solving (Pinto and Teixeira 2020) and developing robust graduate attributes in engineering students who are likely to work in a culturally diverse workplace. However, simply increasing student numbers in engineering programmes is not enough. There is a need to consider who drops out, why, and how they could be supported. The South African context is further hampered by the legacy of racial discrimination, injustices, inequalities and unequal socio-economic and educational structures that continue to favour middle-class students (De Clercq 2020).

As higher education expands to accommodate more students, it is imperative to understand our current students and what their journeys thus far have entailed for us to tailor an educational experience that best suits them. In this way we can provide "a foundation for continuous improvement" (Maller et al. 2007, 3). A knowledge of the profiles of our students will allow us to examine and adjust our strategies and pedagogies to serve the diverse learning and engagement needs of all our students through the development of directed interventions. Reflecting on teaching in a scholarly and directed way will lead to a refinement of the learning activities to meet the actual rather than perceived academic needs of students and go some way towards addressing the diversity in classes. Resulting curriculum changes to enhance the provision of mathematics support have the potential to improve retention and graduation rates by providing education that caters for all students.

The study of mathematics is vital to engineering courses and has an important bearing on the success of engineering students. However, many concerns have been raised by researchers about the transition from studying mathematics at high school to university (Kouvela 2017; Solomon and Croft 2016). In addition, large classes and diversity in earlier academic experience make the teaching and learning of mathematics in higher education a difficult terrain to navigate (Lithner 2011), particularly for students who are second-language English speakers as they must "fight two enemies" (Brock-Utne 2000, 185): the language and the disciplinary content. We agree with Martin (2006, 202) that "teaching and curriculum design can contribute to … racialized experiences and … disparate achievement and persistence", and that good teaching and reforming the curriculum will not alone overcome "the murkiness and complexity of socio-contextual forces".

Engineering programmes at many South African institutions are competitive, accepting only students with the best school results. Yet retention of engineering students is an ongoing concern in South Africa (Mouchou Tchamdjeu, Steenkamp, and Nel 2018). This study examines data from the University of Cape Town (UCT). Despite being the top ranked university in Africa (CWUR 2020), the UCT graduation rates for undergraduate engineering degrees have remained below the National Plan for Higher Education goal of 20% of incoming students graduating in minimum time (UCT 2018, 187). Progress statistics show that 15% of engineering students who started at UCT between 2010 and 2014 left within five years due to slow academic progress; 8% left in good academic standing, and five-year completion rates are skewed in favour of white students (83%) compared to Indian (69%), coloured (63%) and African (49%) students (UCT 2018, 206–10). Various forms of government-funded extended curriculum programmes exist at South African universities to increase access and success, recognising that the "need to overcome our apartheid past … is an on-going battle" (Garraway 2017, 112).

This research is situated in the Academic Support Programme for Engineering (ASPECT) at UCT. ASPECT is a structured extended curriculum programme that allows students to choose to complete a four-year engineering degree in five years, by spreading the load for the first two years over three years, based on their performance in their first six weeks at university. Students take mathematics and physics in smaller classes where the pedagogy subscribed to is one of active learning. Mathematics and physics lectures and tutorials are rooted in social constructivist theories of learning, are interactive, involve regular and immediate feedback, and centre on cooperative learning groups. Further opportunities for learning are created by enabling study groups to form and by appropriately using technology. A supportive environment is created through scaffolding provided by patient and caring lecturers and tutors with strong discipline and pedagogical expertise.

Having passed two semester courses in engineering mathematics (Calculus 1 on differential calculus and Calculus 2 on integral calculus), the third course, Calculus 3, on vector calculus, remained an obstacle, with failure rates ranging from a modest 17% to a high of 48% in the years 2015 to 2019. Calculus is central to engineering studies and vector calculus is a difficult course in undergraduate mathematics studies (Othman et al. 2015). We have observed that many students struggle with the concepts in this course. There are various issues attributed to this struggle, as suggested by Padayachee and Craig (2020), that include difficulty with visualising and sketching in three dimensions, a lack of problem-solving skills, students' beliefs, and that the course demands mastering complex and new ideas in a limited time.

As mathematics lecturers in this support programme, we reflected on the question, "How can the analysis of student data contribute to understanding student performance in calculus?" In this research article we seek to identify the key factors influencing calculus results. To achieve this end, we apply statistical methods, focusing on the relative importance analysis using university admission data from National Benchmark Tests in mathematics, quantitative literacy, and academic literacy. Data was analysed from students whose first attempt of Calculus 3 was in one of the 10 semesters in the five years from 2015 to 2019.

The variables highlighted as having the highest bearing on students' performance in Calculus 3 were academic literacy subdomains, irrespective of home language spoken. The unexpected findings led to purposeful reflections on how the analysis of student data can contribute to our scholarship of teaching and learning and how important it is to avoid subconscious assumptions about students' backgrounds and their impact on student success. The findings will be interpreted using Moschkovich's (2013, 2015) framework of academic literacy in mathematics, which is discussed in relation to broader academic literacy theory after the next section on the value of statistical methods to answering our research question.

Finding the Most Important Factors that Explain Calculus Results

Relative importance analysis (RIA) is used as a supplement of a statistical method called multiple regression, which measures how changes in one variable affect another variable (Tonidandel and LeBreton 2011). The goal of RIA is to better understand the extent to which each predictor variable in a regression model contributes towards explaining variance (R-squared) in the predicted variable. The predicted variable in this study is Calculus 3 scores. This understanding is achieved by breaking down the R-squared into parts such that the value of each part is the contribution of each predictor variable towards R-squared. The method to partition the model's R-squared is not straightforward in data where the predictor variables (for example, scores in three admission tests) are correlated (Stadler, Cooper-Thomas, and Greiff 2017). Two RIA methods to determine the importance of variables have emerged in the literature: dominance analysis and relative weight analysis (Tonidandel and LeBreton 2011). The main purpose of a dominance analysis is to examine patterns of importance of variables, giving a list of the main variables that influence variation. Relative weight analysis provides information on how much a predictor variable contributes towards the predicted variable either by itself or in combination with the other predictor variables. The RIA method used for this article is relative weight analysis. The analysis provides insights about the contribution made by each predictor variable towards the predicted variable that aids in building theory around the relationship between Calculus 3 performance and performance in admission tests. Even though the method has faced a few criticisms, relative weight analysis helps to better understand the relationship between several predictors in a regression model and the predicted variable. The results from the analysis could be used to strengthen or challenge existing theories about a specific subject area and aid theory building on the subject area under investigation (Tonidandel and LeBreton 2011), which in this research is student performance in engineering mathematics.

Academic Literacies in the Engineering Curriculum

Academic literacy is the ability to use language to meet the demands of tertiary education across different fields of study (Weideman 2018). A disciplinary literacy pertains to reading, writing and communicating within a specific discipline (Srivastava 2017), and is defined as knowledge that supports students' understanding of concepts related to a field of study, such as mathematics (Stoffelsma and Spooren 2019). Weideman (2018) contends that academic literacy, seen as the ability to use language for cognition and analysis, is fundamental in the progression of a student's education, which is negotiated through language. Cognition and analysis play a significant role in a student's mathematics education. Accuracy of meaning in mathematics is requisite and therefore understanding mathematics texts, which usually have "a dense presentation style" (Shanahan, Shanahan, and Misischia 2011, 422), is paramount. Students also need to be "inducted" into mathematical discourse and allowed to develop their accuracy of mathematical language use. Navigating the symbols and diagrammatic representations in mathematical texts requires students to manoeuvre between academic mathematical language, school language and home language. Special attention should be given to academic literacy in mathematics for many reasons: the texts are complex and difficult and require extra effort to process, concepts are dense and abstract, terminology used is likely to be unfamiliar to students and not encountered in everyday language use, the specialised vocabulary, symbols and diagrams place high demands on students, and there is a need to recognise and understand patterns, infer main ideas and use both inductive and deductive reasoning (Stoffelsma and Spooren 2019). In their research on understanding approaches to student writing, Lea and Street (1998) present three theories: a "study skills" perspective, an "academic socialisation" perspective and an "academic literacies" perspective. Focusing on grammar, syntax, punctuation, and spelling, the study skills perspective views writing as a set of learnt skills transferrable across disciplines. The academic socialisation perspective was initially thought of as inducting students into academic culture, but has been critiqued for the assumption that the university has one culture and for ignoring diversity. Academic socialisation is now viewed as the induction of students into disciplinary or subject-based cultures and norms (Lea and Street 2006).

Embracing the evolving nature of disciplinary cultures and norms and encompassing both the study skills and academic socialisation perspectives, an academic literacies perspective views literacies as social practice where the different ways of knowing and identity are valued as influencing student learning. Importantly, the academic literacies perspective acknowledges that students must negotiate different academic literacies within the university and "the requirement to switch practices between one setting and another, to deploy a repertoire of linguistic practices appropriate to each setting, and to handle the social meanings and identities that each evokes" (Lea and Street 1998, 159).

Moschkovich (2015, 44) adopts a sociocultural approach to academic literacy in mathematics (ALM) by assuming that mathematical activity is mediated by language, signs and social activity, by focusing on the potential for progress rather than learner deficiencies or misconceptions, and by extending the ALM framework beyond simply "language as words" and "mathematics as numbers". We align with Moschkovich's (2015) definition of academic literacy in mathematics as three-stranded and intertwined, encompassing mathematical proficiency, mathematical practices, and mathematical discourse. Mathematical proficiency (Kilpatrick, Swafford, and Findell 2001) is described as the combination of five interdependent strands: (1) conceptual understanding, which involves understanding concepts, operations, relations and comprehending connections and similarities between interrelated facts; (2) procedural fluency, which is flexibility, accuracy and efficiency in implementing appropriate procedures and includes when and how to use procedures; (3) strategic competence, which is the ability to formulate and solve mathematical problems; (4) adaptive reasoning, which is the capacity to think logically about concepts and conceptual relationships required to navigate through various procedures and concepts to solutions; and (5) a productive disposition, which includes the positive self-perceptions that develop as students gain more mathematical understanding and capacity to do mathematics. Moschkovich's interpretation of mathematical proficiency is proficiency in the content of mathematics, which can be summed up as understanding, representing and reasoning in mathematics. Mathematical practices are sociocultural interactions in a community of practice (Lave and Wenger 1991) that involve mathematical thinking and the use of signs (Moschkovich and Zahner 2018), and include problem-solving, generalising, sense-making, reasoning, abstracting, using, connecting and looking for patterns in structure or regularity, modelling and imagining (Moschkovich 2015). Mathematical discourse connects mathematical cognition to sociocultural practices and allows for participation in mathematical practices. More than just the use of language, mathematical discourse involves being able to use valued mathematical tools such as abstracting, generalising, and using arguments that are precise, brief and well-defined (Moschkovich 2015), and it is where students actively transform learning through joint activity. Discourse is the essential element to their participation in mathematical activity and "involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages and the everyday register)" (Moschkovich and Zahner 2018, 1000).

Methods

A quantitative analysis was used to explore the mathematical proficiencies of engineering students at a residential South African university, focusing on the academic preparedness of students and to identify groups of students to target for follow-up qualitative research. Diagnostic information from the National Benchmark Tests (NBT) was used to explore students' entry level mathematical proficiency. Engineering students' mathematics performance in the three calculus courses was analysed.

The research participants were 10 cohorts of engineering students (731 in total) in an extended curriculum programme whose first attempt at the 12-week engineering mathematics course, Calculus 3, was in one of the semesters in the period 2015–2019. A minimum score of 50% allows enrolment in the next calculus course, with failed courses repeated in the following semester. First-attempt scores from the two prerequisite engineering mathematics courses, Calculus 1 and Calculus 2, and for Calculus 3, were gathered for analysis.

Students self-identified their "race" as defined under the previous apartheid system and home language. Gender options were limited to male and female (which is no longer the case). The research cohort was 86% South African. The language groups were represented as English (44%) and other languages (56%). There were 66% male students and 34% female students. The "race" differentiation was 48% black African, 17% coloured, 10% white, 12% Indian/Asian, and 13% unknown due to students choosing not to identify a "race" category.

To assess the difference in the performance of Calculus 3, performance groups were created from both Calculus 1 and Calculus 2 based on the standard university performance grouping: Fail (0–49%); 3 (50–59%); 2- (60–69%); 2+ (70–74%); 1 (75–100%). The "Fail" group includes students who did not take the final examination due to achieving less than the minimum score.

One-way analysis of variance (ANOVA), chi-square tests and relative importance analysis were used within the different performance groups to assess the differences in Calculus 3 performance. An ANOVA was used to assess whether the differences between the means of the Calculus 3 performance groups were statistically significant.

Results

The analysis shows that there were statistically significant differences within the performance groups with each of the demographic variables. The ANOVA showed that there was a significant difference ($p = 0.0024$) between the mean of the male students (54.8%) and the female students (58.6%), with a standard deviation of 15.8% and 15.5%, respectively, in Calculus 3.

A statistically significant ($p = 0.0155$) difference was found between the Calculus 3 scores of English home-language (EHL) students (54.5%) and English second-language (ESL) students (57.4%), with standard deviations of 16.5% and 15.1%, respectively. Citizenship and population groupings showed no statistically significant differences in Calculus 3 scores.

First-attempt performance in Calculus 1 had an effect on Calculus 3 performance. As might be expected, the high performers in Calculus 1 were the high performers in Calculus 3, and high performers in Calculus 2 were also high performers in Calculus 3.

Further to the analysis by demographic variables, ANOVA showed that the performance in Calculus 3 was significantly influenced by the Calculus 1 performance. Participants in the top performance group (based on Calculus 1 scores) had the highest mean score of 67% in Calculus 3, with mean scores dropping to 61%, 56%, 52% and 51% in the 2+, 2-, 3 and Fail categories in Calculus 3, respectively.

The pattern was similar when repeating the analysis using Calculus 2 performance groups. Participants in the top performance group of Calculus 2 had the highest mean score of 68% in Calculus 3; the 2+ performance group had a mean score of 57%; the 2- category had a mean score of 55%, and participants who were in the 3 or Fail performance groups each had the lowest mean score of 47% in Calculus 3.

Tracking Progress with Relative Importance Graphs

Extending our focus to explore the possible influence of pre-university diagnostic data on performance in Calculus 3, we analysed results from the National Benchmark Tests (NBT), which cover 20 subdomains across tests in mathematics (5 subdomains), quantitative literacy (6 subdomains) and academic literacy (9 subdomains, see Appendix). Using the same performance groups (1, 2+, 2-, 3, Fail), a multiple regression analysis was used to determine the independent variables that appear to be more important in explaining the performance in each Calculus 3 performance group. The NBT subdomains were used in the multiple regression analysis as independent variables and each of the Calculus 3 performance groups as the dependent variable. First, the relative importance analysis was done on the whole group and then on the Calculus 3 results grouped according to performance in Calculus 1 and Calculus 2. The graphs and the discussion below relate to the results of each analysis.

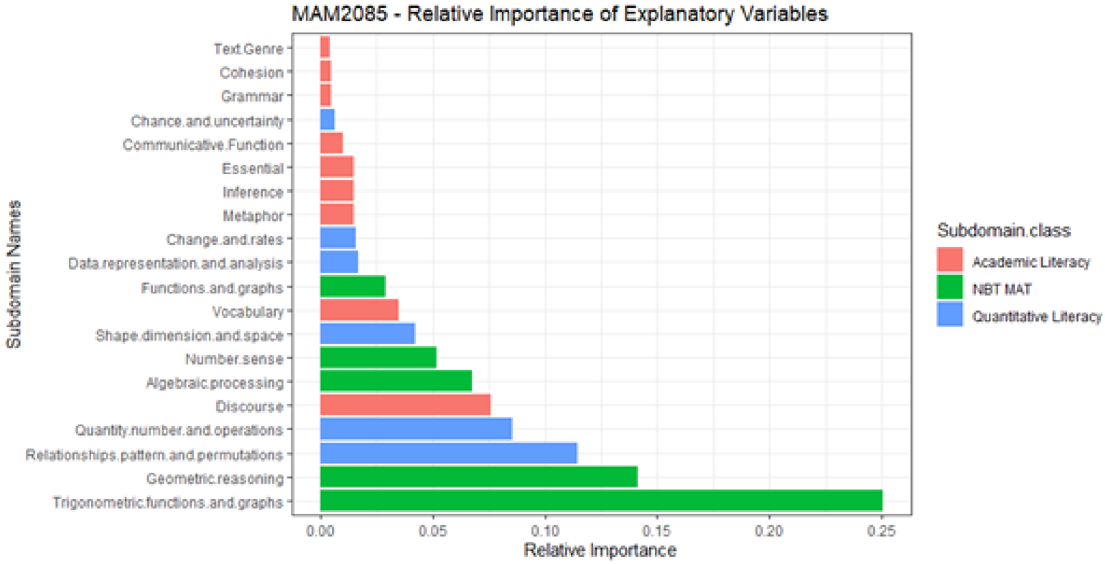

Figure 1: Relative importance of NBT explanatory factors on overall Calculus 3 performance

When analysing ungrouped Calculus 3 scores for all 10 cohorts, the NBT variables that explained most of the variability in Calculus 3 were the mathematics subdomains of *trigonometric functions and graphs* and *geometric reasoning*, and the quantitative literacy subdomain of *relationships, pattern and permutations*, as shown in Figure 1. The importance of prior mathematics for further mathematics is in line with expectations for a discipline with a hierarchical structure, where new concepts build on earlier concepts. Interestingly, the academic literacy subdomain *discourse* accounts for a greater explanation of variance of all Calculus 3 students than do the mathematics subdomains of *algebraic processing* and *number sense*, which are important prerequisites for the study of calculus. However, the pattern does not hold when the analysis is repeated for subgroups according to performance in Calculus 3.

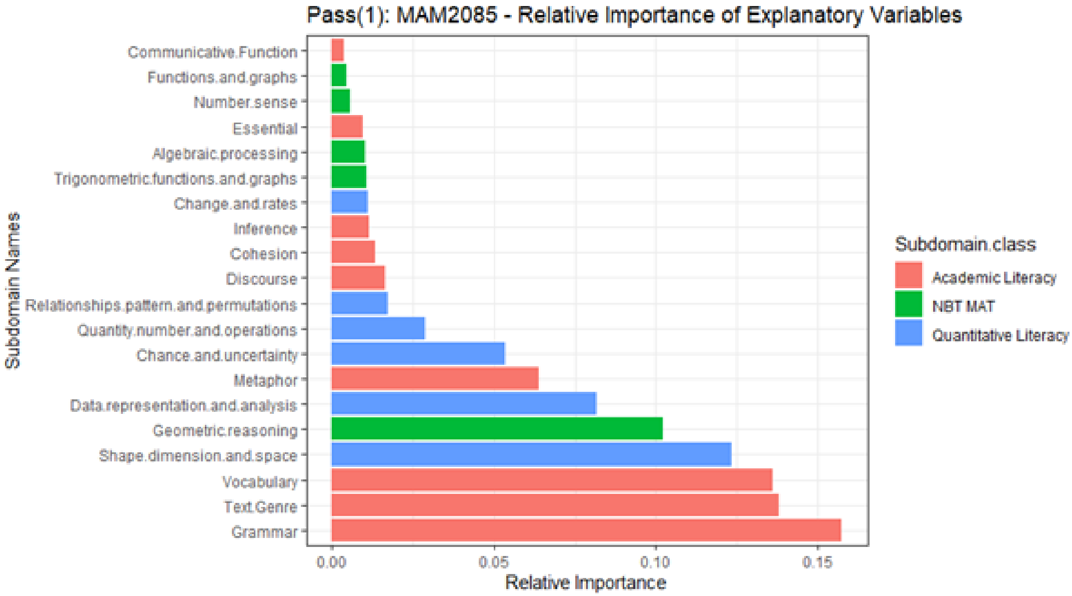

Figure 2: Relative importance of NBT explanatory factors on Calculus 3 performance of group 1

When the analysis is repeated with both Calculus 1 and Calculus 2 subgroups, it is revealed that academic literacy subdomains account for the largest percentage in explaining the variance in Calculus 3 performance in the highest achieving category (group 1). *Grammar*, *text genre*, *vocabulary* and *inference* are the highest appearing subdomains.

Similarly, looking at the highest performing group in Calculus 3 based on only Calculus 1 performance, we note that the top explanatory variables are the academic literacy variables of *grammar*, *text genre*, and *vocabulary*, as shown in Figure 2.

A similar pattern emerges when analysing the Calculus 3 results of the highest performers in Calculus 2. The factors with greatest relative importance are not from mathematics subdomains but from academic literacy (*grammar* and *inference*) and quantitative literacy (*change and rates*, and *quantity, number and operations*). In summary, the Calculus 3 scores of high-achieving students are explained more by academic literacy variables than other NBT variables.

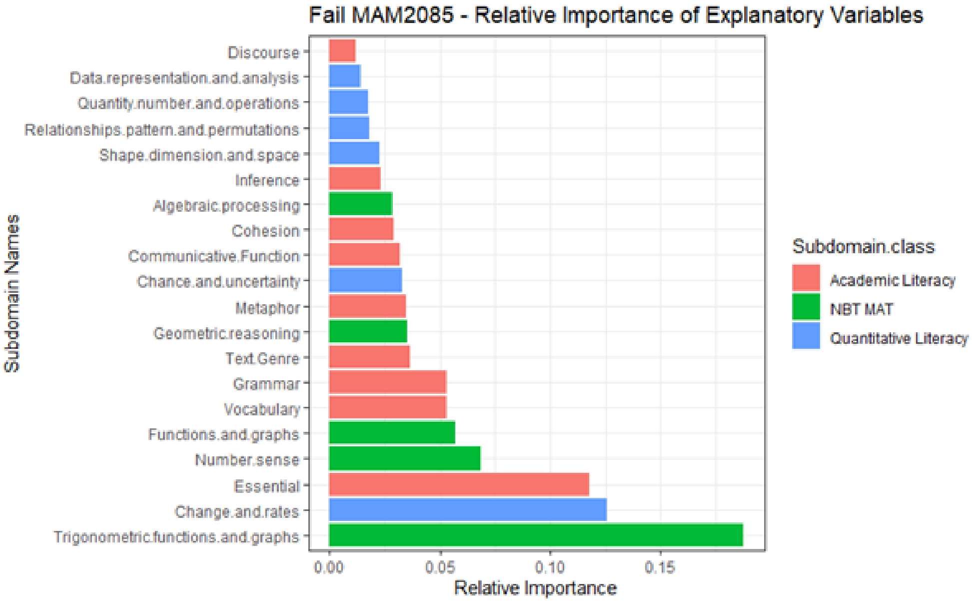

Figure 3: Relative importance of NBT explanatory factors on Calculus 3 performance of Fail group

Contrastingly, the most important variables that explain the Calculus 3 performance of students in the Fail category are in the mathematical and quantitative literacy subdomains, followed by academic literacy subdomains, as shown in Figure 3.

Comparing Academic Literacy Subdomains for Five Performance Groups

In seeking to understand better the relative importance analysis correlation patterns of the performance groups, we investigated the box and whisker plots for these groups for each academic literacy subdomain. Since the subdomains of *cohesion*, *communicative function* and *discourse* are important elements in mathematical procedure (Sango and Steyn 2020), we expected these subdomains to have the greatest relative importance in explaining Calculus 3 marks.

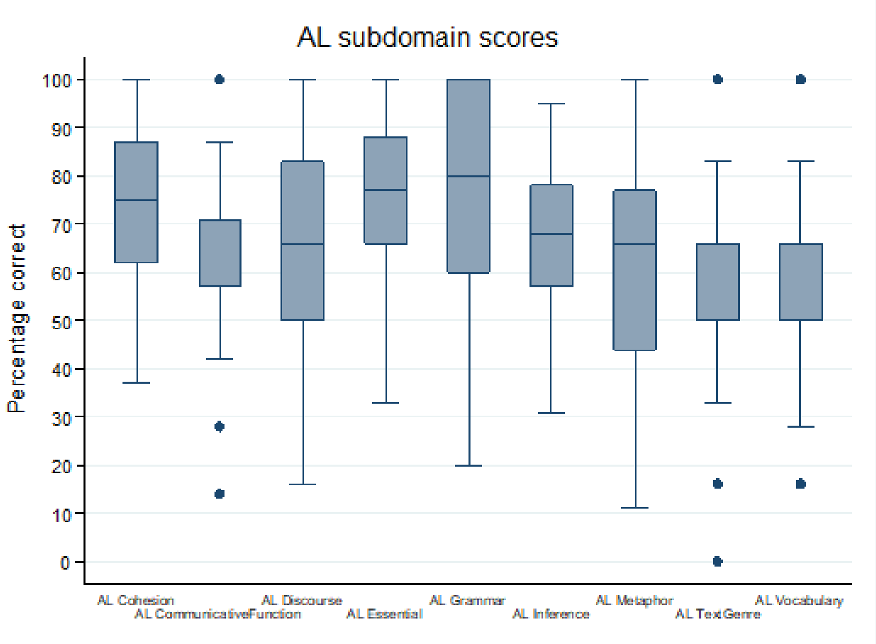

Figure 4: Box and whisker plot of performance on NBT Academic Literacy (AL) subdomains of overall cohort

As seen in Figure 4, the performance of the whole group in all the academic literacy subdomains was relatively high. Notably, in the Fail category there were several outliers below 50% and the score ranges were smaller than for other groups.

Text genre and *vocabulary* were the subdomains in which students performed the worst across all the performance groups, with more than 75% of high achievers attaining scores below 65%. Surprisingly, there was little difference in performance in the *text genre* and *vocabulary* subdomains between students in the Fail or top performing categories of Calculus 3 scores. In addition, the short box plots indicate a strong agreement in scores for these subdomains between students.

The *grammar* subdomain showed more variation between the Fail and 1 performance groups. Although both groups showed three quarters of the students scoring above 60%, the high performers had half of the students concentrated between scores of 60% and 80%. Surprisingly, the Fail and 3 categories showed half of the students scoring above 80% while the other categories had only a quarter of the students scoring above 80%. The Fail group had the tallest box plot for *grammar*, an indication of the large variation in *grammar* scores for that group.

Cohesion displayed a similar pattern across all performance groups, except for group 1 where a quarter of the students scored below 50%. The performance of the other groups appeared to be better, where a quarter of the students scored below 65%.

Communicative function showed almost identical trends across all groups. The subdomain *metaphor* showed tall box plots, which is an indication of a wide variation of scores, except for the Fail group where the box plots were short and there were many outliers. The *discourse* subdomain shows similar trends for all categories except, again, for the Fail group where there were many outliers.

Discussion

We reflect on the data analysis using the definitions of the NBT academic literacy subdomains and the three categories in Moschkovich's (2015) framework for academic literacy in mathematics—mathematical proficiency, mathematical practices, and mathematical discourse—to address our research question.

Mathematical Proficiency Progression

In this section, the focus is on students' performance in Calculus 1 and Calculus 2, and the impact of their performance in Calculus 3. Those with Calculus 2 scores below 60% were more likely to fail than to achieve the passing score (50%) for Calculus 3. Students who pass Calculus 2 but fail Calculus 3 would not have met all the strands of mathematical proficiency that are needed for mathematical understanding, reasoning and representing and therefore their capacity to succeed in further mathematics would be compromised. Students can pass a course by achieving high marks in one section and low marks in another, potentially creating gaps for courses that build on the neglected topics. Students in transition from Calculus 2 to Calculus 3, especially those with results of 50–59%, may be helped by interventions that address the issue of uneven engagement with course content by requiring sub-minimum achievement in all topics in a course. A more modular approach would signal the importance of proficiency in each topic for effective engagement in mathematical practice and mathematical discourse.

Strengthening Conceptual Understanding in Mathematical Practice

Based on the Calculus 2 and Calculus 3 performance data, low-performing students in Calculus 2 are more likely to have inadequately developed mathematical proficiency and may be less able to articulate this in the required mathematical practice or to engage in mathematical discourse and communication in Calculus 3.

Mathematics is considered a challenging discipline and the challenge is often exacerbated by excessive workloads and time pressures to cover the concepts prescribed by a particular course. Teaching, assessment and learning are often linked to traditional, non-interactive, teacher-centred approaches, with the student positioned more as a receiver of teacher-directed knowledge than an active participant (Helstad, Solbrekke, and Wittek 2017). Such traditional teaching typically presents fewer opportunities for students to engage in mathematical practices. Moschkovich (2013, 49) suggests that "[i]nstruction should provide opportunities for students to engage in mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical concepts, communicating their thinking, justifying their reasoning, and critiquing arguments”. These practices are especially important for the development of a conceptual understanding of mathematics (Dunnigan and Halcrow 2021), but the fast-paced curriculum followed in Calculus 1 and 2 does not lend itself to ideal mathematical practices. Similarly, in Calculus 3 students are expected to master complex concepts in a short time in a busy engineering schedule with a full workload and little time to revisit any gaps that may have resulted from previous courses.

Moschkovich (2015) calls for a balance between conceptual understanding and procedural knowledge in the quest to connect and integrate these two types of knowledge. It may be argued that Calculus 1 and Calculus 2 place more emphasis on procedural fluency and less on conceptual understanding. In many cases, the emphasis on procedures caters to other disciplines such as physics and chemistry that need these procedures for the teaching of their concepts. Unfortunately, this is done to the detriment of engaging in the mathematical practices necessary for deep conceptual understanding. This imbalance plays itself out negatively when students encounter Calculus 3, where conceptual understanding of new and cognitively more demanding concepts is central to their success in the course. Moreover, instruction should encourage opportunities for students’ active communication and meaning-making using mathematical language, such as mathematical practices involving reasoning, constructing arguments, expressing structure and regularity and solving problems (Moschkovich 2013).

Mathematical Discourse and Success in Calculus 3

It is interesting to note that ESL students’ performance in Calculus 3 (mean = 57%) was better than that of the EHL students (mean = 55%) and is statistically significant ($p = 0.0155$). At first glance, this result may be unexpected, but it fits with the definition of mathematical discourse as more than only the use of “technical words with precise meanings, but rather the communicative competence necessary and sufficient for competent participation in mathematical discourse practices” (Moschkovich 2013, 46). Separate from general language proficiency, which typically favours home-language students, mathematical discourse also includes using “valued mathematics tools such as abstracting, generalising and using arguments that are brief, precise and well defined” (Moschkovich 1999, 12) that are not likely to be developed outside a mathematics class.

There are distinct differences in the relative importance analysis of the highest performing and lowest performing groups in Calculus 3. We infer from the relative importance analysis that, for the highest performing Calculus 3 students, high achievement in the NBT academic literacy subdomains of *grammar*, *text genre* and *vocabulary* may play a prominent role in their navigation of mathematical text when initially engaging with concepts and when answering new questions. For the lowest performing students, we note that, in addition to having to decode the language, weaknesses in other domains play a more prominent role. As a result, the lower performing students are most likely to have issues with other fundamental concepts in mathematics and quantitative literacy in addition to academic literacy.

We take note that ESL students are at a disadvantage, having to contend with academic literacy conventions in a language they may not be proficient in (Sibomana 2016). However, our findings show that academic literacy proficiency is important for academic success in Calculus 3 for all students, both EHL students and ESL students.

The compelling case made by the data analysis of this quantitative research study alerted us to the significant role that academic literacy plays in the success of engineering mathematics students in a support context. We acknowledge that, although illuminating, this is only an initial view and that future qualitative research will reveal further nuances. This quantitative study is an exemplification of the potential of data to influence pedagogy and to impact student learning.

The relative importance of academic literacy in explaining mathematics results is much stronger in Calculus 3 than anticipated. We suggest that the level of difficulty of the mathematics in Calculus 3 is greater than in the preceding two engineering mathematics courses and speculate that when the mathematics is less cognitively challenging and more formulaic weaknesses in academic literacy do not prevent students from achieving course requirements and progressing to the next level of their degree. The compounding effect of a higher level of difficulty of the mathematics (Pearce et al. 2015) together with weak academic literacy may explain why some students who pass the first two calculus courses are less successful in Calculus 3.

Traditional mathematics curricula and pedagogies are often adhered to, citing "It has always been done that way" as a justification. Consequently, steering away from traditional norms to innovative ways of teaching and learning is infrequent. As educators of mathematics, we realise that we must continuously strive to engage in pedagogies that move away from simply requiring our students to reproduce knowledge and instead engage them in activities that support critical thinking, reading, writing and discussion of mathematical concepts.

Conclusion

Students who enter tertiary studies with less developed academic literacy may need more directed instruction to develop their academic literacy that the current system offers, where they are expected to "pick up" skills through the variety of course materials and tasks required in each of their courses. The findings of this quantitative analysis confirm to us what other research studies have reported—that we should be more targeted in developing students' academic literacy, particularly in first-year mathematics courses, so that students are more likely to succeed with more cognitively demanding courses such as Vector Calculus as their education progresses. Furthermore, this research highlighted the method of relative importance analysis used to drill down deeper to show the weighted importance of each variable on Calculus 3 performance.

This article supports a simultaneous focus on the three components of academic literacy in mathematics (Moschkovich 2013), especially for extended curriculum programme students. The next steps in our research will be to explore how to implement changes to our teaching of mathematics in ways that support academic literacy development from within our discipline. We will draw from studies that favour discipline-based academic literacy curriculum interventions, such as Carstens (2016) and Goodier and Parkinson's (2005) argument that language use developed in context is likely to be more effective than a generic language development course.

Future research should assess the effectiveness of our curriculum interventions informed by the relative importance analysis with regard to academic literacy development. We suggest that such an assessment should adapt Ally and Christiansen's (2013) rubric for teachers to assess the opportunities for students to develop each of the mathematical proficiency strands and include Moschkovich's (2015) two further strands of *mathematical practices* and *mathematical discourse*. By assessing our provision of opportunities to develop mathematical proficiency, mathematical practice and mathematical discourse (Moschkovich 2015), rather than only assessing how students take up the opportunities, we can avoid simply blaming students for not developing.

The emphasis of support programmes in engineering often does not include academic literacy development within disciplines. This research provides evidence that the success of students in engineering mathematics is dependent on their proficiency in academic literacy and argues that this component be integrated within the discipline in the support environment. We concur with Kelly-Laubscher and Van der Merwe (2014, 1) that without interventions, many students are "unlikely to gain access to disciplinary ways of learning and writing, which ultimately may lead to their exclusion from university". We echo the sentiment of Sibomana (2016) that the time has come for all institutions of learning to empower their students with academic literacy within their disciplines so that they may succeed.

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Appendix

The National Benchmark Test Academic Literacy Construct (Sango and Steyn 2020)

Skill Assessed	Explanation of Skill Area
Cohesion	**Identify links** (anaphoric and cataphoric) and other **mechanisms that connect** parts of text.
Communicative function	**Identify and understand function** of parts of **sentences / discourse**: define; exemplify; support/endorse; contradict; or persuade etc.
Discourse relations	**Understand** the **structure and organisation of discourse and argument**: 1) transitions in argument, logical development; 2) how main idea, supporting ideas / 'evidence' are presented; 3) introductions / conclusions.
Essential / non-essential (Distinction making)	**Make distinctions, classify / categorise and compare:** identify main ideas and supporting detail; cause and effect; statements and examples; facts and opinions; propositions and their arguments; being able to 'label'
Grammar / syntax	**Understand and analyse** the **grammatical and sentence structures** in academic language, and how that **affects / can change meaning**
Extrapolation, application & inferencing	Draw **conclusions** and **apply insights**, either on the basis of what is **stated** in texts or is **implied** by these texts.
Metaphorical expression	**Understand and work with metaphor in language.** This includes the capacity to perceive language connotation, word play, ambiguity, idiomatic expressions, and so on.
Text genre	Can **perceive 'audience'** in text, **purpose** in writing and understand **register** and **tone**: includes an ability to understand text register (formality / informality) and tone (didactic / informative / persuasive / etc.)
Vocabulary	**Derive/work out the meaning of words/terms** from the context