Article

Examining Learners' Understandings of Algebraic Variables: Evidence from Modelling in the Classroom

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Abstract

In response to difficulties in learning algebra, researchers claim that the modelling approach is a powerful tool to build a foundation for learning mathematics at the middle school level. This article explores the conceptual difficulties and understandings of variables that Grade 8 learners have when they work in a modelling activity after these concepts have been learned from Grades 5–8. To this end, 25 Grade 8 learners in one school (in one classroom) in North Wollo district were conveniently sampled to participate in this qualitative methods study. In this study, data were collected using audio recordings of communicative events and analysed using learners' conversational statements during their discussion to solve the modelling activity. The findings in this study indicate that participating learners' understandings of algebraic concepts (variables, constructs and sub-constructs) after these concepts were taught using the traditional method of teaching from Grades 5–8 did not equip them with the necessary skills to understand variables as unknown quantities involved in real-life problem situations. This study provides an insight for teachers to incorporate modelling in their classrooms by exploring the strategies in the textbook to explain concepts in mathematics through the lens of Action, Process, Object, Schema theory. It could also serve as a catalyst for further investigation on the effectiveness of the teaching treatment implemented in this study in other domains of mathematics with the aim of helping learners develop a sound conceptual understanding.

Keywords: modelling approach; problem-solving instruction; model-eliciting activity; algebra; genetic decomposition

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Introduction and Background

Scientists use modelling to visualise and organise their thinking into simpler representations to be shared and communicated to their peers. Not surprisingly, modelling as an instructional strategy benefits learners in connecting and developing their knowledge when they learn mathematical concepts. According to Shahbari and Peled (2017), the importance of integrating modelling into primary school mathematics is twofold: It provides an opportunity for learners to fully engage in the construction of mathematical models and at the same time to strengthen their mathematical knowledge. In addition, learner-generated models have great utility in revealing learners' difficulties and their conceptual understanding and give an opportunity for teachers to provide rapid, individualised, and specific feedback to promote both better modelling and conceptual understanding (Wilson et al. 2020). A modelling approach to the teaching and learning of algebraic concepts, algebraic variables in this study, focused on the mathematisation of realistic situations that are meaningful to learners.

In response to difficulties, specifically in learning algebra, Ferrucci et al. (2008) claim that a modelling approach to teaching and learning is a powerful tool to improve problem-solving skills among middle school learners. They pointed out that this approach helps learners to visualise abstract mathematical relationships in the form of a model through which learners can gain a deeper understanding of the concepts and skills to manipulate symbols as variables. They further recognised the modelling approach as a critical algebraic solving competence. According to these scholars, there are two phases in instruction that uses the modelling approach. The first involves examining relationships among variables in the situation. In the second phase, with a series of mathematical transformations, a model expressed in terms of symbolic expressions, graphs or tables is proposed (Ferrucci et al. 2008).

In the context of Ethiopia, however, such a learner-centred approach to instruction is not common. Several studies characterised instruction in Ethiopian classrooms as teacher dominated (e.g., Desta et al. 2009), with no active involvement of learners (Desta et al. 2009; Weldeana 2016) and no collaborating groups, where learners are passive rather than active receivers of knowledge (e.g., Gulfo and Obsa 2015) and there is no inclusion of real-life context in the teaching process (e.g., Weldeana 2016). Even though the government of Ethiopia has called for an active teaching-learning approach, its implementation appears to be superficial and inefficient at school level (Gulfo and Obsa 2015; Serbessa 2006). The traditional method of instruction in the classroom results in several consequences for learners' learning. As a psychological component, learners' beliefs regarding learning mathematics are affected to the extent that they do not realise that learning can take place without their teacher talking and writing in the classroom (see Buishaw and Ayalew 2013; Desta et al. 2009; Tiruneh 2020). Furthermore, mathematics is viewed by many learners in Ethiopia as having "little" application in the real world (e.g., Weldeana 2016). Worse still, many learners think they are "slow" in mathematics (Weldeana 2016, 146). With regard to primary school learners' achievements, almost all national learning assessment processes conducted in Ethiopia for the last two decades have shown that learners' achievements, particularly in mathematics, have declined dramatically (MoE 2018 cited in Bati 2020). Relative to norms set internationally, for example the Trends in International Mathematics and Science Study (TIMSS), about 50% of the Ethiopian learners at the age of 12 fail to reach the low achievement benchmark for learners aged 10 years (Singh 2014). As pointed out by the Ministry of Education (MoE 2018), an ineffective teaching approach and a focus on quantity at the expense of quality, among others, are reasons for such a decline. Therefore, there is an urgent need in Ethiopia to conduct a study that implements a modelling approach (an innovative approach) to learning and teaching, which is usually seen as self-inviting and supports learners in self-directed learning and development (Blinkston 2000).

Model-Eliciting Activities (MEAs) are characterised by a strong thought-revealing aspect, meaning that teachers' observations of learners' modelling activities provide them with a rich source of information about learners' knowledge (Lesh et al. 2000) and help them to strengthen learners' understanding of the mathematical concepts embedded in the MEA. Kaiser and Sriraman (2006) stated that "an MEA is a problem-solving activity constructed using specific principles of instructional design in which learners make sense of meaningful situations and invent, extend, and refine their own mathematical constructs" (Kaiser and Sriraman 2006, 306). Lesh and Sriraman (2005) recognised MEA as a medium not only to promote critical algebraic thinking in learners, but also to develop a more comprehensive understanding of mathematical concepts embedded in the MEA. In this study, a Model-Eliciting Activity (MEA) that is called "an authentic real-life problem" was used to guide the design of the teaching sequence for the teaching treatment.

Law (2008) points out that deep conceptual understanding takes place when the learner is provided with opportunities to reflect on her problem-solving activities in a meaningful learning situation. However, according to Cai (2003), less is known about questions such as, "What are the ways that are used by learners when they attempt meaningful learning of mathematics through modelling and problem-solving?" Furthermore, other scholars in the field of modelling and problem-solving called for more research to shed light on the role of modelling and problem-solving tools to achieve concept development in mathematics (Sriraman and English 2010) and to assess the mathematical skills that caused difficulties in the learning of specific concepts (Moll et al. 2016). More research is also required to accumulate knowledge on modelling and problem-solving instruction as research in this regard has been slower to emerge (Lester 2013). Other scholars also pointed out that although much of the mathematics we use today was developed as a result of modelling real world situations, modelling as an instructional strategy is not well represented in school mathematics curricula (Wilson et al. 2020).

Much research has been reported abroad around learners' understanding of algebraic notation (e.g., Novotná and Hoch 2008), learners' understanding of literal terms and expressions (e.g., Kieran 2007), and on systematic mistakes that learners make in simplifying algebraic expressions (e.g., Linchevski and Herscovics 1996). Several researchers have revealed that the learning of algebra for many learners is difficult. Many of the difficulties in learning algebra come from learners' poor understanding of two important concepts, namely the variable and algebraic expression (Banerjee and Subramaniam 2012). Successful learning of algebra is important as the findings of several researchers disclosed that most difficulties in learning mathematics in later grades are predominantly related to learners' poor understanding of basic algebraic concepts (e.g., algebraic variables) in earlier grades (e.g., MacGregor 2004). Following MacGregor's argument, it can be said that learners' poor performance in mathematics in later grades is influenced by their knowledge of algebra. This argument is particularly evident as algebra and all high-level mathematics courses are built based on the concept of the variable.

The content domains in mathematics at elementary school level (Grades 1–8) in Ethiopia include four thematic areas. These are numbers, algebra, geometry, data handling and chance (probability). Specific to algebra, learners are introduced to the use of letters to represent numbers and are taught to find the value of a variable or an expression by substituting letters for numbers in Grades 5 and 6. Learners learn to compare expressions to determine whether they are equivalent or not in Grades 7 and 8. They also learn to identify variables in real-life situations with real-life word problems and solve using linear equations in Grades 7 and 8 with an aim to develop knowledge in various specific domains in mathematics to solve problems encountered in their day-to-day life activities (MoE 2018). Therefore, learners in Grade 8 in Ethiopia have encountered all constructs and sub-constructs of algebraic variables in algebra set for upper elementary levels (Grades 5–8). As one of the four content domains, the researcher in this study regarded algebra as one of the most important areas of school mathematics. But despite its importance, little attention (if any) has been given and less is reported in the literature in the context of Ethiopia about learners' understandings and their difficulties with algebraic variables. This study has been proposed to shed light on the role of a modelling approach in the teaching and learning of algebraic concepts, in particular the algebraic variable.

The primary goal of the modelling approach adopted in this study was to observe the learning opportunities created by the designed teaching sequence to broaden learners' understandings of the concept of variables when they fully engage to solve a modelling activity in a group setting. Furthermore, the study also examined the way in which participating learners perceived variables as unknown quantities. To this end, this study made use of Leinhardt, Zaslavsky, and Stein's (1990) characterisation of variables. According to Leinhardt, Zaslavsky, and Stein, variables such as time, length (e.g., distance, height), speed, temperature, weight, age, and number of people are considered contextualised variables (Leinhardt, Zaslavsky, and Stein 1990). The focus of this study was the number of people as a contextualised variable (Bestgen 1980; Krabbendam 1982) that can be involved in the situation of a problem. This study also used the suggestion made by Ursini and Trigueros (2001) on the knowledge required by learners to understand variables as unknown quantities. According to Ursini and Trigueros, to understand a variable as an unknown quantity, learners should be able to 1) recognise and identify the presence of something unknown in the context of a problem that can be determined by considering the restriction of that situation, 2) symbolise quantities involved in the situation of the problem, 3) interpret symbols (letters) that appear in equations as the representatives of specific numbers, 4) substitute symbols in equations with numbers to create relations that are true numerical statements, and 5) determine unknown quantities that appear in equations by performing suitable algebraic and arithmetic operations (Ursini and Trigueros 2001).

As clearly noted, the objectives of the content and sub-content of algebra set for primary schools for Ethiopian learners are aligned with the suggestions made by Ursini and Trigueros (2001). Therefore, this study used these suggestions to examine the way in which participating learners perceived variables as unknown quantities as a result of their learning from Grades 5–8 using the traditional method of teaching. Specifically, the study addressed the following research question:

To what extent are groups of Grade 8 learners able to use their knowledge of variables when they engage to solve a model-eliciting activity?

Theoretical Background

The modelling teaching sequence employed in this study explored two main theories, namely the Models and Modelling Perspective (MMP) and the Action, Process, Object, Schema (APOS) theory.

The MEA used in this study was designed based on the MMP to serve as a medium of instruction aimed at developing the conceptual understanding of learners in algebra. The six design principles used as guidelines for the design of an MEA are based on the Modelling and Modelling Perspective (Lesh et al. 2000), as explained below: 1) The reality principle requires the activity to help the learner to be able to interpret the situation in the given problem. 2) The model construction principle requires the activity to help the learner to be able to interpret mathematically the quantities, relationships, and patterns that they need to take into account. 3) The self-evaluation principle provides an opportunity for learners to evaluate their responses and those of others and to assess the adequacy of the responses. 4) The documentation construction principle requires learners to prepare a written document while working on the activity to demonstrate their own thinking on the problem situation. 5) The model generalisation principle requires learners to generalise the mathematical model in such a way that another person at the same level of competency could apply it to a similar situation. 6) The simple prototype principle requires the activity to be as simple as possible, which enables learners to produce a reasonable solution.

The modelling teaching sequence used in this study was designed based on APOS theory. APOS is an acronym for Action, Process, Object, Schema (Dubinsky and McDonald 2001). Dubinsky and McDonald explain that APOS theory assists in the comprehension of various concepts in mathematics instruction in order to understand the process of learning by providing clarification on learners' various cognitive activities during problem-solving and the construction of knowledge in mathematics instruction (Dubinsky and McDonald 2001). The theory involves the following aspects:

Action: An action conception is the transformation of a concept perceived by the learner as essentially external. A learner with an action conception can perform the transformation based on learned, detailed steps.

Process: When actions are repeated and interiorised, the actions collectively become a *process* (Breidenbach et al. 1992). That is, an internal representation of the same actions is constructed in a learner's mind, but not necessarily with extra stimuli.

Object: A learner is said to possess an object conception when she is able to view a process as consisting of a collection of single elements. We say that the encapsulation of processes into an object occurred (DeVries and Arnon 2004) when learners apply rules in a specified sequence that goes beyond the ability to use them for calculation.

Schema: According to Dubinsky (2000), the three aspects mentioned above and other schemas organised in a structured way are necessary to have complete knowledge of a concept.

Genetic decomposition (Arnon et al. 2014) is a hypothetical model that involves the following: the theoretical analysis of the action conception that the learner performs explicitly, the process conception in which the learner imagines taking an action, the object conception, which is where the learner sees a concept as a single element, and the schema conception, in which the learner applies the previous conceptions.

In this study it was expected that (as a result of learning the various algebra concepts set for primary school level from Grades 5–8) learners would possess some level of the necessary background knowledge on the concepts of variables upon which the processes of learners' previous learning in their previous schooling can be judged. The background knowledge in this study constitutes the action conceptions of algebraic variables. These conceptions are a set of mental constructs that might describe how the concept of variables as unknown quantities can develop in the mind of a learner. The constructs proposed in this paper are:

- The number sentence, and
- The concept of variables.

A lack of knowledge on any of the above-mentioned concepts may cause serious challenges for learners when they examine relationships among variables in the MEA as part of their solving processes of the designed task; it may also cause problems in terms of their understandings of variables as unknown quantities, as suggested by Ursini and Trigueros (2001). In order to answer the research question, the data collected during the *action conception stage* based on APOS theory were used to examine learners' knowledge on variables as part of the required knowledge to solve the MEA. An intern helped the researcher to describe the learning opportunities created by the designed teaching sequence in relation to learners' understandings of the concept of variables as a result of learning through the traditional method of teaching in their previous schooling.

Methodology

The Sample, Sampling Technique, Research Setting and Method

The sample for the study comprised Grade 8 learners (*n*=25) from one classroom consisting of 53 learners in an elementary school. For research employing purposeful sampling, Bernard (2002) recognised the importance of incorporating availability and willingness to participate in selecting participants. Out of 53 learners, 25 were purposely sampled based on their willingness to participate in an out-of-school time intervention. Learners in Grade 8 were the focus of this study because by the end of Grade 8, learners in Ethiopia have encountered all constructs and sub-constructs of variables in algebra set for upper elementary levels (Grades 5–8). In addition, learners sit for primary school leaving examinations at the end of Grade 8. The participating learners were divided into five groups each with five members, as recommended by English and Walters (2005) for modelling activities. The groups were formed by the participants' mathematics teacher to ensure that the groups were heterogeneous in terms of the learner's mathematical achievement. Each group engaged in the designed MEA (garden activity) for three hours in a week, one hour a day. To solve the given MEA, learners should be able to do the following: 1) recognise and identify the presence of something unknown in the context of the MEA that can be determined by considering the restriction of that context; 2) categorise the identified quantities involved in the context of the MEA as unknown quantities and/or constants; 3) identify unknown quantities and/or constants that are relevant to solve the given MEA out of the various quantities involved in the context of the MEA; 4) symbolise relevant quantities involved in the context of the MEA using any letter of their choice; 5) use the symbols (letters) that represent the relevant quantities to develop an equation (model) that enables them to solve the given MEA; 6) interpret letters that appear in the developed model as representatives of specific numbers; 7) substitute letters in the developed model with numbers to create relations that are true numerical statements; 8) determine values of the identified relevant unknown quantities that appear in the developed model by performing suitable algebraic and arithmetic operations; 9) consider an expression as a single entity from a set of algebraic expressions; 10) recognise a variable (algebraic expression) that can be substituted for another variable (expression) in which both describe the same situation; and 11) describe a different situation so that the developed model works. Ursini and Trigueros (2001) suggested that five from the above list are necessary to understand a variable as an unknown quantity.

In this study a qualitative method was employed. Participant learners (in a group of 5) were closely observed to assess the learning opportunities created by the MEA (designed by the researcher) and to analyse learners' difficulties that arose during their work to tackle the MEA. Exploring learners' skills while solving a problem helped the researcher to explain what learners do and do not understand in terms of the algebraic concepts (e.g., variables) relevant to the learning of algebra.

Data Collection Instruments

The Modelling Approach Teaching Treatment

In this study a modelling approach was used as an instructional strategy that involves learners in developing their competences in algebra using the designed MEA. To this end, a combination of APOS and MMP theories was used as a framework in this study. The learners worked on the MEA in groups to explore the context of the MEA, to actively engage in processing one another's ideas to make sense of the MEA, and to negotiate the meaning to start doing the MEA. They wrote their answers on a common worksheet as they were guided by the researcher through the various stages of conceptions, mainly in the action conceptions stage. Learners' modelling activities in groups were audiotaped to transcribe their discussions (conversational statements).

Data Collection Procedures and Method of Data Analysis

The difficulties learners encountered related to conceptions of variables were analysed by identifying learners' conversational statements dealing with difficulties from the transcripts. The statements were those dealing with difficulties that arose during the implementation of the teaching treatment that learners were able to resolve through the guidance of the researcher. The groups were audiotaped and their discussions were transcribed and analysed. Due to space constraints, the findings are discussed based on the analysis of the discussions of two groups (Group A and Group B).

The teaching treatment was implemented from April 2020 to May 2020 by the researcher. One reason the researcher implemented the teaching treatment was that such a teaching method was completely new to Grade 8 teachers in the participating school. As mentioned, 25 learners participated in the teaching treatment. The negotiation between the researcher and authorities of the school gave rise to an agreement to allocate three hours per week, one hour per day, to conduct the teaching treatment during out-of-school time for practical reasons. The role of the researcher was to guide learners through the various stages of the preliminary genetic decomposition. In each stage of the preliminary genetic decomposition, the researcher intervened in learners' solving processes of the MEA to encourage them without eliminating their struggle in solving the task.

Findings

With an eye to answering the research question, the researcher was focused to find evidence of learners' understanding of variables from their conversational statements. In this study, the learning episodes in the processes of tackling the given MEA, mainly at the action conception stage, require learners' understandings of the concept of variables as unknown quantities. The focus of this study was the action conception stage of the teaching sequence as learners' understandings of variables can be described completely at this stage, as suggested by Ursini and Trigueros (2001).

To this end, this study made use of Leinhardt, Zaslavsky, and Stein's (1990) characterisation of variables and the suggestion made by Ursini and Trigueros (2001) to understand variables as unknown quantities. Therefore, in this study, the data collected during the action conception stage of the APOS theory were used to examine learners' background knowledge on variables at the level of Grade 8. In the following sub-sections, the analyses of the discussions of two groups (Group A and Group B) are presented due to space constraints. A letter and number, for example A1, indicate the group (Group A) and the learner in the group. Similarly, B3 would indicate the group and the third learner in group B.

Difficulty in Recognising Variables in the Problem Situation

In responding to the given MEA (garden activity), learners need to recognise the presence of unknown quantities in the context of the MEA. It was observed that the participating learners in Grade 8 had limited understanding of variables in this sense, which was exhibited in their modelling process, as shown below in the conversational statements.

> Researcher: From your selected quantities in the MEA there are quantities which are variables (unknown quantities) and constants.
> A1: I cannot see any variable in the problem?
> A3: It is also not clear for me.
> A5: There is no x or y in our problem. Where are the variables?
> A2: There are variables such as x or y only in an expression or equation.

From the discussion, learners in both groups seem to explain their understanding of a variable as a letter (symbol) found only in an expression or equation. From this it can be said that learners were unable to recognise and identify the presence of something unknown in a problem situation. According to Ursini and Trigueros (2001), this ability, among others, is necessary to understand the use of a variable as an unknown quantity in a problem situation. To broaden learners' interpretations of variables, the researcher guided the discussion by making use of Leinhardt, Zaslavsky, and Stein's (1990) characterisation of variables. Accordingly, variables such as a number of people are considered contextualised variables (Leinhardt, Zaslavsky, and Stein 1990).

> Researcher: A1, how many learners enrolled in your class?
> A1: There are 48 learners enrolled in our class.
> Researcher: How many learners do you expect to come to school to prepare vegetable gardens in one of the school days next week?
> A1: It will be 48.
> A2: Hmm … How do we know the actual number of learners who will come to school next week?
> A4: There are only 44 learners present in our class today. Four learners are absent today.
> Researcher: A1, what do you say to your friends' explanation?
> A1: … I change my mind and I understand that it is difficult to predict the number of learners in our class who come to school on a particular day.
> Researcher: Such unknown quantities that can be recognised in a problem situation are variables. Quantities that stay the same are constants.
> Researcher: Can you tell me some quantities which are variables or a constant in the MEA?
> A4: Therefore the number of learners in the junior group who will be involved in preparing vegetable beds in a particular day is a variable.
> A5: The same is true for elementary group.
> A2: What about the number of learners assigned to prepare each bed?
> A3: This is clear. We assigned six learners. The number six is constant.

The discussion above and the learning episode of the modelling approach teaching sequence at the action conception stage of APOS theory suggest that the traditional teaching methods being used by teachers in learners' previous schooling (Grades 5–8) did not help them to recognise and identify the presence of unknown quantities in a problem situation. Although the learners who participated in both groups were unable to notice the real situation as it occurs in their classroom, they were prompted by the researcher. Based on Leinhardt, Zaslavsky, and Stein's (1990) view of contextualised variables, the learners in both participating groups recognised variables in the MEA and resolved their difficulties. This study argues that the designed teaching sequence and appropriate guidance from the researcher in this study helped learners to broaden their interpretation of algebraic variables involved in a problem situation.

Difficulty in Representing Unknown Quantities Using Letters

As evidenced from the discussion, learners also encountered difficulties in representing unknown quantities in the given MEA using letters.

> Researcher: Can you please represent the relevant variables you identified in the given MEA with letters of your choice and a constant by number.
> A1: It is not clear teacher.
> Researcher: Can you use a letter in the English alphabet to represent the identified variables in the given MEA?

A4: Let y [stand] for the number of vegetable beds that will be prepared by learners in the elementary and junior groups.
A1: No! It should be b to represent the number of vegetable beds.
A3: It should be v.
A4: There is no difference whether we use y, b or v to denote the number of vegetable beds that will be prepared by learners in both groups.
Researcher: Why?
A4: Y is a letter or symbol that stands for numbers. The same holds true for b and v.
Researcher: A3, do you agree?
A3: Yes I do agree.
Researcher: A1, do you change your mind now?
A1: Yes I do.

This finding is consistent with the mathematics education literature, which contends that learners (like A1 in our case) tend to interpret literal symbols not as representations of numbers but as abbreviated names (i.e., "b stands for bananas") (e.g., Kieran 2007). Kinzel (1999) asserts that learners' difficulties with algebraic notation originate from narrow conceptions of variables. Following Kinzel's argument, it can be said that the traditional teaching methods used by teachers in their previous schooling were ineffective to help learners understand the concept of variables in different contexts. The explanation given by A4 resolved A1's difficulties, as observed above.

Difficulty Interpreting Variables

Although the majority of participating learners were able to represent variables in the given MEA, some group members were unable to interpret variables correctly. This suggests that learners in the group have a habit of memorising procedures or algebraic rules without a conceptual understanding attached to expressions involving variables in mathematics discourse from their previous schooling. This is consistent with the mathematics education literature, which contends that learners mainly have difficulty interpreting symbols when they move from arithmetic (the manipulation of numbers using basic operations) to algebra (the manipulation of variables) (Carraher and Schliemann 2007; Kieran 2007). These difficulties were clear from their discussions, as can be seen below. The explanations given by B1 and B5 resolved learners' difficulties, as observed in the following discussion.

B4: We already denoted y for the number of vegetable beds that will be prepared by learners in the elementary and junior groups.
B1: We also let w stand for the number of learners in the elementary group in a particular school day.
B3: We denote x for the number of learners in the junior group in a particular school-day.
B5: We said 6 represents the group size to prepare a single bed.
B2: We have already described the number of learners in both groups using w+x.
Researcher: We said 6y describes the number of learners in both groups expected to come to the school garden.

B5: Also, we said that w+x describes the number of learners in the elementary and junior groups who come to school on a particular school day.
Researcher: B3, please tell us what we did to put 6y and w+x mathematically.
B3: It is not clear teacher.
B2: Hmm. We used the equal sign.
Researcher: How do you put this equality mathematically?
B5: 6y=w+x.
Researcher: What will 6y=w+x look like if w and x are equal?
B3: How could w and x be equal?
B1: It is also not clear to me. They are different variables.

At this point we see a misconception regarding variables displayed by some of the participating learners (B1 and B3). For these learners, different letters mean different values. As pointed out by Stephens (2005), many learners think $h + m + n = h + p + n$ is never true because "*m*" is different from "*p*" (Stephens 2005). Keeping in mind the contextualised nature of a variable (Leinhardt, Zaslavsky, and Stein 1990), the researcher guided the discussion to broaden the learners' interpretation of a variable.

Researcher: As we know, of course, the number of learners enrolled in the elementary group w (Grades 5 and 6) and that of the junior group x (Grades 7 and 8) are not equal.
Researcher: But, if equal numbers of learners in both groups come to school on a particular school day, what do you say about w and x?
B1: Teacher, still not clear.
Researcher: B2, do you have something to say?
B2: The two variables w and x will be the same.
Researcher: Can you explain using an example?
B5: If, for example, 300 learners in Grades 5 and 6 and the same number of learners in Grades 7 and 8 come to school on a particular day, then w and x are equal.
Researcher: B3, when will x and w be different? Can you explain using an example?
B3: If, for example, 300 learners come in the elementary group and 320 in the junior group.
Researcher: In this case, B1, what do you say about w and x?
B1: Hmm. We say w=x.
Researcher: All of you, do you agree with B1?
Researcher: [They all agree]. Different letters may represent the same value. Therefore, different letters may not mean different values.

Determining the Value of an Unknown Quantity

Researcher: How many vegetable beds will be prepared only by learners in the junior group if they all come to school on that particular school day?
B5: That means we have to ask the number of learners enrolled in the junior classes.
B1: What about learners in the elementary group?
B4: We are asked to use learners only in the junior group.
B2: What do we put for w in our rule?
B3: It must be zero.
Researcher: Take the number of learners in the junior group to be 300.

After a short discussion between the group members, they wrote the answer for the number of vegetable beds prepared by the junior group.

This suggests that the traditional method of teaching mainly relies on procedural skills and rote memorisation. This is consistent with the mathematics education literature, which contends that teachers never moved away from the traditional methods of mathematics instruction (e.g., Gainsburg 2012; Harbin and Newton 2013).

Evidence for the Effectiveness of the Teaching Treatment

From the analysis in the above sub-sections, learners' conceptual understandings of algebraic concepts (e.g., variable constructs) after being taught using the traditional method of teaching in previous schooling (Grades 5–8) seem weak. In a sharp contrast to the traditional method of teaching, the researcher in the modelling approach of teaching guided the discussion through guiding questions and by making use of the contextualised nature of variables (Leinhardt, Zaslavsky, and Stein 1990) to develop a clear conceptual understanding of variable concepts embedded in the given MEA. The overall improvement in learners' understanding as observed from their conversational statements in the action conception stage (of the learning episode) was evidenced from the modelling approach teaching sequence guided by the designed MEA, which describes the learners' mental construction in learning algebra based on APOS theory.

Conclusion

The results revealed that the participating learners who learned algebra through the traditional method of instruction from Grades 5–8 were unable to understand the concept of a variable as an unknown quantity as proposed by Ursini and Trigueros (2001). They failed to recognise the presence of something unknown in the situation of the given MEA. Learners' ability to interpret variables was also limited. Except for two learners in the two focus groups, learners displayed their skills in determining an unknown quantity (variable) that appeared in an equation by performing arithmetic operations, but failed to interpret the answer in relation to the situation of the MEA. This can be said to be a result of their previous experiences. The participating learners seem to depend on rote memorisation when learning mathematics and the teachers who employed the traditional method of teaching seem to teach the learners using rules and procedures in order to get the correct answers, but neglect their conceptual understanding. An analysis of the conversational statements of learners in the two focus groups indicates that the modelling teaching approach, which was employed by making use of Leinhardt, Zaslavsky, and Stein's (1990) characterisation of variables, was superior to the traditional teaching method in demonstrating algebraic concepts. This positive influence was evidenced in the recorded conversational statements made by the learners during the teaching model lesson. This means that the modelling approach teaching treatment has the potential to improve the learners' understanding of algebraic concepts. In exploring learners' skills while solving a real-life problem, the designed MEA helped the researcher to explain what learners do and do not understand in terms of algebraic concepts (e.g., variables and expressions) after being taught using the traditional method of teaching in previous schooling (Grades 5–8). The findings from this study suggest that there is a need to find an alternative teaching method in the teaching and learning of algebra at primary school level to develop a deep conceptual understanding in learners. This study could also serve as a catalyst for further investigation on the effectiveness of the modelling teaching approach in other domains of mathematics with an aim of helping learners to develop a sound conceptual understanding.

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