Article

Decoherence Challenges in Nanoscience: A Quantum Phase Space Perspective

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Abstract

Quantum decoherence is both the fundamental mechanism underlying the quantum-to-classical transition and a major challenge for the development of scalable nanoscale quantum technologies. This work introduces a quantum phase space (QPS) framework that provides a unified geometric description of decoherence. The framework is based on a dual structure consisting of an overcomplete continuous frame of minimum-uncertainty states $|\langle z\rangle\rangle$ that defines the QPS geometry, and an orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ that satisfies the strict mathematical requirements for decoherence within the objectivity criterion of the spectrum broadcast structure for pointer states. Within this framework, the variance–covariance matrix $\mathcal{G}(t)$ of the QPS ground states serves as a universal indicator of decoherence regimes; it is time-independent ($\dot{\mathcal{G}} = 0$) for Markovian (memoryless) dynamics, and time-dependent ($\dot{\mathcal{G}} \neq 0$) for non-Markovian

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dynamics with memory and information backflow. To illustrate the formalism, the Hu–Paz–Zhang master equation is generalised to include simultaneous position and momentum couplings to the environment, leading to a generalised non-Markovian master equation characterised by a spectral-density matrix. The corresponding evolution equations for the first moments ($\langle p(t)\rangle$, $\langle x(t)\rangle$) and for the ground covariance matrix $\mathcal{G}(t)$ establish a direct connection between microscopic environmental properties, classical-like trajectories and the QPS geometry. In the Markovian limit, the QPS geometry becomes time-independent, formalising the concept that a stationary environment selects and freezes a specific set of quasi-classical trajectories. The non-Markovian regime naturally describes memory effects, information backflow, recoherence, and evolving deformations of the uncertainty ellipse. The proposed framework provides a unified theoretical foundation for modelling decoherence in nanoscale systems involving simultaneous position and momentum interactions with the environment, including nanoelectromechanical resonators, optomechanical systems, and superconducting quantum circuits. The QPS framework may therefore bridge fundamental theory and practical quantum engineering, offering a promising coherent pathway to understand, control and exploit decoherence at the nanoscience frontier.

Keywords: quantum decoherence; pointer states; quantum phase space; nanoscience; quantum technologies

1 Introduction

Quantum decoherence is one of the most fundamental phenomena in modern physics. It describes the progressive loss of quantum coherence resulting from the interaction between a quantum system and its surrounding environment, thereby providing the physical mechanism underlying the emergence of classical behaviour from quantum dynamics. Beyond its foundational role in the quantum-to-classical transition and the quantum measurement problem [1]–[9], decoherence has become a central issue in nanoscience and quantum engineering, where it determines the stability, coherence time, and performance of nanoscale quantum devices, including quantum computers, quantum sensors, superconducting circuits, quantum dots and nanoelectromechanical system [10]–[15].

Unlike classical physics, quantum mechanics allows physical systems to exist in coherent superpositions of different states [1]–[4], [16], [17]. Nevertheless, the macroscopic world exhibits well-defined classical properties rather than coherent superpositions. Quantum decoherence provides the physical mechanism that bridges these two regimes. Through continuous interaction and entanglement with its surrounding environment, the reduced density operator of a quantum system progressively loses its off-diagonal coherence terms and becomes approximately diagonal in a preferred basis of so-called pointer states. These environmentally selected states are comparatively robust against decoherence, remaining stable under the system-

environment interaction and thereby giving rise to the emergence of classical-like behaviour [1]–[6], [17]–[25].

During the past decades, the characterisation of pointer states has been driven by complementary paradigms such as environment-induced superselection (einselection) [2], [21], [22], the predictability sieve criterion [23]–[25] Quantum Darwinism [19]–[21], [26], and the spectrum broadcast structure (SBS) [27], [28]. Despite the significant advancements these theories offer, developing a unified and mathematically rigorous framework for decoherence, one that, in particular, accurately captures the identification and dynamical evolution of pointer states in realistic environments, remains an open challenge [18], [19]. In parallel, understanding the role of environmental memory has become increasingly important across quantum physics, from fundamental studies to nanoscale technologies and quantum devices [9], [11]–[15], [29]–[36]. Although Markovian dynamics is now widely recognised as a particular limit of the more general non-Markovian description, a unified geometric framework capable of relating both regimes to the evolution of quantum states is still lacking. In realistic nanoscale systems, finite environmental memory, information backflow, and strong system-environment correlations profoundly influence decoherence [32]–[36], making it desirable to develop a formalism in which both Markovian and non-Markovian dynamics emerge naturally from the same underlying geometric description.

In this work, we address these challenges by introducing a novel theoretical framework rooted in the recently developed concept of quantum phase space (QPS) [36]–[38]. The proposed approach provides a unified and mathematically rigorous description of quantum decoherence based on a geometric representation of quantum states in phase space. The QPS formalism is characterised by a dual mathematical structure: an overcomplete continuous frame of joint momentum-coordinate minimum-uncertainty states, denoted $\{|\langle z\rangle\rangle\}$, analogous to the family of states known as coherent and squeezed states in the literature [4], [6], [36]–[43], and defines the geometric structure of the QPS; and an associated orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ which satisfies the strict mathematical requirements expected of a pointer-state basis. This construction establishes a natural link between QPS geometry, environment-induced microscopic dynamics, and the emergence of classical-like behaviour, ultimately paving the way for a unified geometric framework to describe quantum decoherence.

Leveraging this dual mathematical structure, we show that the QPS serves as a natural point of convergence for all major pointer-state criteria. Under this framework, einselection, Quantum Darwinism, and SBS collectively identify the orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ as the preferred pointer basis. Concurrently, criteria focused on dynamical stability, such as the predictability sieve and minimal entropy production directly single out the "ground state" $|\langle z\rangle\rangle$ as the most classically predictable pure state.

To illustrate the power of this geometric framework and bridge it with established methodologies for microscopic open-system dynamics, we revisit the Hu–Paz–Zhang

(HPZ) master equation, one of the most comprehensive non-Markovian descriptions of quantum Brownian motion [44]–[47]. Generalising the interaction Hamiltonian to encompass simultaneous position and momentum couplings between the system and its environment naturally introduces a spectral-density matrix. This matrix supersedes the conventional scalar spectral density, ultimately yielding a generalised non-Markovian master equation. From this generalised master equation, we obtain explicit closed-form evolution equations for the expectation values pair ($\langle p(t)\rangle$, $\langle x(t)\rangle$) and the covariance matrix (t), directly linking the microscopic properties of the environment to the geometry of the QPS. This generalised framework provides a suitable foundation for modelling decoherence in nanoscale systems where both position and momentum interactions with the environment play a significant role.

The article is structured as follows. Section 2 reviews the foundational principles of quantum decoherence and the role of pointer states in multi-level quantum systems, presenting the four main selection criteria. Section 3 examines the implications of decoherence for particle motion in nanoscience and quantum technologies, emphasising the distinction between Markovian and non-Markovian regimes. Section 4 introduces the QPS framework, establishes the overcomplete continuous frame and the associated time-dependent orthonormal basis, and identifies potential pointer states for particle motion. Section 5 provides an illustrative application of the formalism by presenting the standard HPZ model and extending it to include momentum coupling, deriving the generalised master equation and the explicit evolution equations for expectation values and the ground variance–covariance matrix. Finally, we conclude with a discussion of the broader implications of our approach for quantum technologies, fundamental physics, and the relativistic and cosmological extensions of the QPS framework. Throughout this article, quantum operators are denoted in boldface to distinguish them from their classical counterparts and expectation values.

2 Quantum Decoherence and Pointer States

2.1 General Formalism

Pointer states are the quantum states that remain stable when the system interacts with its environment. They are the states that decoherence does not "destroy" [1]–[9]. Decoherence tends to destroy superpositions through environmental entanglement, but pointer states are the ones that do not become entangled in a way that destroys their classical-like behaviour. They are the states that survive environmental monitoring. We can think of them as the eigenstates of the system observables that are continuously "measured" by the environment [1]–[5], [21], [22]. A first way to technically define pointer states is then to consider them as the eigenstates (or approximate eigenstates) of the system operators that the environment continuously monitors. Because the environment acts like a measuring device, it selects a preferred basis in which superpositions rapidly decay. This basis is formed by the set of pointer states and is then called the pointer basis [1]–[3], [21]. Suppose the interaction Hamiltonian is:

$$\mathbf{H}_I = \sum_k \boldsymbol{A_k} \otimes \boldsymbol{B_k} \quad (1)$$

where operators $\boldsymbol{A_k}$ act on the system and $\boldsymbol{B_k}$ on the environment. Pointer states $|s_n\rangle$ satisfy:

$$\boldsymbol{A_k}|s_n\rangle \approx a_{nk}|s_n\rangle \quad (2)$$

The relation (2) means that pointer states are regarded as approximate eigenstates of the operators $\boldsymbol{A_k}$ that couple to the environment:

- for an environment that measures momentum, the pointer states are approximately momentum eigenstates;
- if the environment measures position, then pointer states are localised position states;
- if it measures energy, pointer states are energy eigenstates; and
- if it measures a spin axis, pointer states are spin states along that axis, etc.

The evolution of the density operator offers another viewpoint for defining and visualising both the emergence of pointer states and the decoherence phenomenon. Consider the system starting in a pure state $|\psi_S\rangle$ and the environment in an initial state $|e_0\rangle$. Suppose the interaction between the system and environment begins at this initial time ($t = 0$). For $t > 0$, entanglement arises between the state of the system and that of the environment. The entangled global state $|\Psi_{SE}\rangle$ can be written in the following form:

$$|\psi_{SE}\rangle = \sum_n C^n(|s_n\rangle \otimes |e_n\rangle) \quad (3)$$

where $\{|s_n\rangle\}$ is a basis in the system state space and $|e_n\rangle$ the basis of the environment state space correlated with $\{|s_n\rangle\}$. The canonical density operator which describes the system environment entanglement is then:

$$\boldsymbol{\rho}_{SE} = |\Psi_{SE}\rangle\langle\Psi_{SE}| = \sum_n \sum_m C^n C^{m*} (|s_n\rangle \langle s_m| \otimes |e_n\rangle\langle e_m|) \quad (4)$$

The density operator $\boldsymbol{\rho}_S = \boldsymbol{\rho}$ of the system can be obtained by taking the partial trace over the environment degree of freedom: $\boldsymbol{\rho} = Tr_E(\boldsymbol{\rho}_{SE})$. Explicitly:

$$\boldsymbol{\rho} = \sum_k \langle e_k|\boldsymbol{\rho}_{SE}|e_k\rangle = \sum_n \sum_m \rho_m^n |s_n\rangle\langle s_m| \quad (5)$$

With

$$\rho_m^n = \sum_k C^n C^{m*}(\langle e_k|e_n\rangle\langle e_m|e_k\rangle) = \langle s_n|\boldsymbol{\rho}|s_m\rangle \quad (6)$$

$\{|s_n\rangle\}$ is the pointer basis of the system if, for the environment basis $\{|e_n\rangle\}$ correlated with it, we have the following relation:

$$\langle e_n|e_m\rangle \simeq \delta_m^n = \begin{cases} 1 \; if \; n = m \\ 0 \; if \; n \neq m \end{cases} \text{for } t \gg \tau_D \tag{7}$$

in which τ_D is called the decoherence time: it is the characteristic time scale after which decoherence effects become significant. We have then

$$\rho_m^n(t) \simeq \sum_n \sum_m C^n C^{m*} \delta_n^k \delta_k^m = \begin{cases} |C^n|^2 & if \; n = m \\ 0 & if \; n \neq m \end{cases} \quad for \;\; t \gg \tau_D \tag{8}$$

which is equivalent to writing for the system's density operator:

$$\boldsymbol{\rho}(t) \simeq \sum_n \rho_n^n \, |s_n\rangle\langle s_n| = \sum_n |C^n|^2 |s_n\rangle\langle s_n| \qquad for \quad t \gg \tau_D \tag{9}$$

Relations (7), (8) and (9) are equivalent and describe the phenomenon of decoherence, which consists of a rapid decay of the off-diagonal terms of the density matrix for $t > \tau_D$. Indeed, these terms represent the existence of coherent superpositions among pointer states. For $t \gg \tau_D$, the matrix representation of the density operator in the pointer basis becomes approximately diagonal. The density operator then becomes an incoherent combination (statistical mixture) of pointer states: the diagonal elements $\rho_n^n = |C^n|^2$ simply represent classical-like probabilities of finding the system in a defined pointer state.

2.2 Criteria for Selection of Pointer-States

The identification of the pointer basis is a central task in the theory of decoherence. While the general formalism presented in the previous section provides the conceptual framework, several independent criteria have been developed to determine the pointer states rigorously. However, these criteria should be regarded as complementary, as they approach the problem from different physical perspectives: structural, dynamical and informational. Moreover, as we will show in Section 4, they converge within the QPS framework. The main criteria are outlined below.

Environment-Induced Superselection (Einselection)

Einselection, introduced by Zurek [2], posits that the environment continuously monitors a set of observables of the system through the interaction Hamiltonian. The pointer states are the approximate eigenstates of the system operators that couple to the environment [2], [21], [22]. This criterion corresponds to the one previously considered in the preceding section to define pointer states via relations (1) and (2). Assuming the interaction Hamiltonian is given by relation (1), the pointer states $|\boldsymbol{s_n}\rangle$ are those that satisfy the condition that they are eigenstates of the operators $\boldsymbol{A_k}$ in the limit where the self-Hamiltonian of the system is negligible. More generally, when neither the self-Hamiltonian nor the interaction dominates, the pointer states arise from a compromise between self-evolution and interaction, and they become localised in phase space. In the opposite limit, where the dynamics is dominated by the system's self-Hamiltonian,

einselection produces pointer states that coincide with the energy eigenstates of the self-Hamiltonian. The environment therefore acts as a measuring apparatus that continuously monitors the observables. States that are not eigenstates of these observables become entangled with the environment and lose coherence. The pointer states, by contrast, remain stable because they are minimally perturbed by the monitoring process. This criterion selects the pointer basis structurally, based on the form of the interaction and the relative strength of the self-Hamiltonian.

Predictability Sieve (Minimal Entropy Production)

The predictability sieve, introduced by Zurek *et al*. [23], takes a dynamical perspective. It identifies pointer states as the pure states that minimise the loss of predictability, that is, the increase in Von Neumann entropy, during the decoherence process [23]–[25]. For an initial pure state $|\boldsymbol{\psi}\rangle$, let $\boldsymbol{\rho(t)}$ be its reduced density matrix after evolution to time $\boldsymbol{t}$($\boldsymbol{\rho}(0) = |\boldsymbol{\psi}\rangle\langle\boldsymbol{\psi}|$). The Von Neumann entropy

$$\mathcal{S}(t) = -\boldsymbol{Tr}[\boldsymbol{\rho}(t)\boldsymbol{ln}\,(\boldsymbol{\rho}(t))] \tag{10}$$

measures the loss of purity due to entanglement. The pointer states are selected by the minimisation of this entropy over all initial pure states with the requirement that the answer be robust when varying $\boldsymbol{t}$ within a reasonable range. Equivalently, one can minimise the decrease of purity $\boldsymbol{P} = \boldsymbol{Tr}[(\boldsymbol{\rho})^2]$ since the loss of purity is directly related to the increase of Von Neumann entropy. This criterion asks: "If I prepare the system in a pure state, which initial state remains the most predictable (ie least entangled with the environment) after decoherence?" The answer selects the states that produce the least entropy or, equivalently, lose the least purity. It is a dynamical criterion that depends on the full evolution, including the system Hamiltonian and the bath correlation functions.

Quantum Darwinism

Quantum Darwinism, developed by Zurek, explains the emergence of classical objectivity through a Darwinian mechanism: the environment selectively proliferates information about certain system observables, creating multiple redundant copies [19]–[21], [26]. The pointer states are those whose information is redundantly encoded in the environment, allowing multiple observers to independently infer the system's state without perturbing it. The term "Darwinism" reflects the analogy to biological natural selection: pointer states are the "fittest" states; they survive environmental monitoring and leave numerous "descendants" by imprinting redundant copies of their information onto environmental fragments. The hallmark of Quantum Darwinism is the mutual information plateau: for any sufficiently large environment fragment $\boldsymbol{F}$, the quantum mutual information between the system $\boldsymbol{S}$ and the fragment $\boldsymbol{F}$ is nearly equal to the Von Neumann entropy of the system:

$$\mathcal{I}(S:F) \simeq \mathcal{S}(S) \tag{11}$$

Here, $\boldsymbol{I(S:F) = \mathcal{S}\ (S) + \mathcal{S}\ (F) - \mathcal{S}\ (SF)}$ is the quantum mutual information, which measures the total correlations between the system and the fragment. The entropy of the system is ($\boldsymbol{\mathcal{S}\ (S) = -Tr[\rho_S ln(\rho_S)]}$). The plateau indicates that the fragment contains almost all the information about the system that is available in the whole environment. The pointer basis is the basis in which the system's state becomes approximately diagonal after decoherence:

$$\boldsymbol{\rho_S \simeq \sum_n p_n |s_n\rangle\langle s_n|} \tag{12}$$

The environment records multiple, independent copies of information about the system's pointer states. This redundancy ensures that different observers who measure different parts of the environment will agree on the system's state, a hallmark of classical objectivity. This criterion is informational: it quantifies how much information about the system is present in environmental fragments. The correlations between the system and the environment may retain some quantumness (quantum discord) that prevents perfect local readout [30]. Quantum Darwinism therefore provides a sufficient condition for the emergence of objectivity but not a necessary structural one.

Spectrum Broadcast Structure

Spectrum broadcast structure, formalised by Horodecki et al. [27] and further developed by Korbicz [28], provides a stronger, structural formulation of objectivity. While Quantum Darwinism focuses on the amount of redundant information (mutual information plateau), SBS specifies the form that this redundancy must take to ensure perfect classical readout. A state is said to exhibit SBS when the environment can be partitioned into fractions, each containing sufficient locally accessible information to perfectly distinguish the system pointer states. The global state $\boldsymbol{\rho_{SE_1 \ldots E_N}}$ of the system and $\boldsymbol{N}$ environment fragments, $\boldsymbol{E_1}$, …, $\boldsymbol{E_N}$, has SBS if it takes the specific tensor product form

$$\boldsymbol{\rho_{SE_1 \ldots E_N} = \sum_n p_n\, |s_n\rangle\langle s_n| \otimes \rho_{E_1}^{(n)} \otimes \ldots \otimes \rho_{E_N}^{(n)}} \tag{13}$$

Equivalently, for any environment fragment $\boldsymbol{E_k}$, the reduced state is

$$\boldsymbol{\rho_{SE_k} = \sum_n p_n\, |s_n\rangle\langle s_n| \otimes \rho_{E_k}^{(n)}} \tag{14}$$

where $\{|s_n\rangle\}$ is an orthonormal pointer basis satisfying $\boldsymbol{\langle s_n|s_m\rangle = \delta_{nm}}$, and the conditional states $\boldsymbol{\rho_{E_k}^{(n)}}$ $(k = 1, \ldots, N)$ of each environment fragment satisfy

$$Tr\left[\rho_{E_k}^{(n)} \rho_{E_k}^{(m)}\right] = 0 \; for \; n \neq m \tag{15}$$

This condition ensures that an observer accessing a single environment fragment can unambiguously identify the system's pointer state. Consequently, each environmental

fragment independently carries complete and locally accessible classical information about the pointer state. The global state exhibits a broadcast structure in which the classical information associated with the system's pointer state is redundantly encoded in multiple, conditionally independent environment fragments. As a result, multiple observers can independently infer the system's pointer state by measuring their respective environment fragments without disturbing either the system or one another, and they will necessarily reach the same conclusion. Spectrum broadcast structure therefore provides a rigorous criterion for objective classical reality. Moreover, every SBS state satisfies the redundancy criterion underlying Quantum Darwinism, whereas the converse is not generally true: redundancy of mutual information alone does not guarantee the existence of an SBS.

3 Particle Motion Decoherence in Nanoscience and Quantum Technologies

Quantum decoherence in particle motion refers to the loss of quantum coherence in the spatial or motional degrees of freedom of a particle due to environmental interactions [1]–[4], [6], [15], [48], [49]. At the nanoscale (approximately between 1 nm = 10^{-9} m and 100 nm = 10^{-7} m), this process fundamentally determines whether particle motion exhibits quantum behaviour (superposition, interference) or classical trajectories. The distinction between Markovian (memoryless environment) and non-Markovian (memory retaining environment) decoherence processes is critical to accurately modelling, controlling and harnessing quantum motion in nanodevices [29]–[35], [50], [51].

3.1 Markovian and Non-Markovian Regimes in Nanoscale Particle Motion Decoherence

In nanoscience, particle motions, whether of electrons, excitons, nanoparticles or nanomechanical resonators, are governed by quantum mechanics but are highly susceptible to decoherence[15], [35], [48]–[53]. The spatial superposition of a particle's wavefunction decays due to coupling with environmental degrees of freedom (phonons, photons, charge fluctuators, defect, impurities, etc) [9], [15], [35], [49], [50], [54]. The timescale and nature of this decay depend critically on whether the environment is Markovian or non-Markovian [29]–[31], [35], [50], [54]. Understanding these regimes guides material choice and device design.

Markovian Decoherence in Particle Motion

Under conditions of weak coupling, high temperature, and a broadband environment (for example, electron–phonon scattering in mesoscopic conductors), decoherence is approximately memoryless, ie Markovian. The off-diagonal elements of the reduced density matrix for a particle in a spatial superposition of separation ($\Delta x = |x - x'|$) decay exponentially:

$$\rho(x,x',t) = \langle x|\rho(t)|x'\rangle = \rho(x,x',0)e^{-[\Gamma.t.(\Delta x)^2]} \quad (16)$$

where $\Gamma = \frac{2M\gamma\, k_B T}{\hbar^2}$ is the constant decoherence rate, M is the particle mass, γ is the friction coefficient, k_B is the Boltzmann constant and T is the temperature. This Markovian description applies, for instance, to:

- electron transport in quantum dots at $T > 1K$, where electron scattering provides rapid environmental relaxation; and
- nanomechanical resonators in low pressure gas environments (P ~ 1 – 100 Pa) where frequent gas collisions create memoryless damping [17].

Non-Markovian Decoherence in Particle Motion

In contrast, when the environment possesses a structured spectrum, finite correlation times, or strong system-environment coupling, decoherence becomes non-Markovian. The off-diagonal elements of the density matrix in the position representation are then described by

$$\rho(x,x',t) = \langle x|\rho(t)|x'\rangle = \rho(x,x',0)e^{-[\Gamma(t).(\Delta x)^2]} \quad (17)$$

In contrast to equation (16), the decoherence function $\Gamma(t)$ in equation (17) is time-dependent and need not increase monotonically. Its non-monotonic behaviour may lead to partial recoherence, reflecting the backflow of quantum information from the environment to the system, which is a characteristic of non-Markovian dynamics.

3.2 Quantum Technologies and Motional Qubits

Quantum technologies exploit coherent particle motion for information processing, sensing and communication (motional qubits). The coherence time of motional qubits, whether charge, phononic or mechanical, is ultimately limited by decoherence, whose Markovian or non-Markovian character dictates control strategies. Markovian regimes permit simple exponential models and conventional dynamical decoupling. Non-Markovian regimes require advanced techniques: reservoir engineering, tailored pulse sequences that synchronise with backflow, and error–correction adapted to non-exponential decay.

Charge Qubits in Quantum Dots

A charge qubit is encoded in the orbital occupancy of a single electron shared between two coupled quantum dots. Decoherence is primarily dominated by charge noise, while electron–phonon interactions contribute mainly to energy relaxation. At sufficiently high temperatures, or when the environmental correlation time is much shorter than the qubit dynamics, decoherence is commonly modelled using the Markov approximation, leading to approximately exponential dephasing ($\tau_D \simeq 10^{-9}s$) [15] [52]. However, at low

temperatures ($T < 100\ mK$), the noise spectrum becomes structured, revealing non-Markovian signatures that must be accounted for in high fidelity control [35] [54]–[57].

Nanomechanical Qubits

A nanomechanical resonator's centre of mass motion can, in principle, be prepared in a spatial superposition. Decoherence arises from coupling to thermal phonons, radiation and defects. In the Markovian regime (for example, moderate vacuum, $T > 100\ mK$), energy decay is exponential and described by a constant quality factor Q. In the non-Markovian regime (cryogenic, high strain conditions), interaction with two level systems leads to complex decay profiles and memory effects, which have an impact on sensing performance and quantum state lifetime [48]-[50], [58]-[62].

3.3 Challenges Related to Decoherence in Particle Motion

Understanding and controlling decoherence is pivotal for exploiting the quantum properties of nanoscale particle motion. The journey from fundamental characterisation to practical mitigation involves navigating a series of interconnected challenges, particularly when environments exhibit memory (non-Markovian dynamics). These challenges span experiment, engineering and theory defining the current frontier of the field, as discussed below.

Characterising Decoherence Regimes

Determining whether particle motion in a given nanoscale device undergoes Markovian or non-Markovian decoherence is non-trivial. Characterisation typically relies on measurements of coherence dynamics, environmental correlation times, and spectral properties, together with theoretical reconstruction of system-environment correlations [29]–[31], [63], [64], and employing coherence witnesses to detect the survival of quantum coherence in these complex environments [65].

Modelling and Simulation

Non-Markovian dynamics are computationally demanding to simulate. Developing efficient numerical methods, such as the reduced hierarchy equation approach and non-Markovian quantum jump techniques, is essential to accurate device prediction [33], [53], [66].

Mitigation in Non-Markovian Environments

While Markovian decoherence may often be reduced using dynamical decoupling, reservoir engineering, feedback cooling, or, where applicable, quantum error–correction techniques, non-Markovian decoherence requires environment aware strategies [15], [54]: engineering bandgaps to suppress emission, using defect free materials, or exploiting memory effects for enhanced sensing and quantum control.

Harnessing Non-Markovianity

A paradigm shift is emerging: rather than suppressing all decoherence, non-Markovian memory effects and the associated backflow of quantum information can be harnessed to achieve quantum advantage [33], [34], [55], [67]. Structured environments can be engineered to enhance quantum performance or enable spatiotemporal control of quantum devices.

Scalability with Motional Qubits

As quantum devices scale, cross talk between motional degrees of freedom and collective decoherence channels may become increasingly important. Managing these in both Markovian and non-Markovian contexts is critical to multi-qubit processors and sensor arrays [11], [15], [33], [68].

Mathematical Physical Framework for Pointer States Description

A major theoretical and practical challenge is the identification of pointer states. In particle motion, these correspond to localised positions or well-defined motional states that are least susceptible to environmental decoherence [17], [18], [23]–[25]. A rigorous mathematical framework capable of describing environment-induced superselection and the emergence of pointer states in both Markovian and non-Markovian regimes would provide a unified foundation for understanding decoherence. Such a framework would directly address the challenges of characterisation, mitigation and scalability by establishing a systematic roadmap for preserving quantum coherence in nanoscale particle motion. In the following sections, we show that an approach based on QPS theory offers a promising route towards this objective by providing a single geometric representation in which coherence decay and pointer-state selection arise as complementary aspects of the same underlying dynamics.

4 Quantum Phase Space Perspective on Decoherence

The challenges outlined in the previous sections, notably the identification of pointer states and the characterisation of Markovian and non-Markovian decoherence, highlight the need for a unified theoretical framework. In this section, we introduce a formalism rooted in the concept of QPS, which addresses these challenges by providing a geometric and mathematically rigorous description of decoherence in particle motion.

4.1 Pointer States for Particle Motion

According to decoherence theory, when a particle's motion is subjected to environmental effects, the interaction results in a selection of pointer states $|\alpha\rangle$ that form the pointer basis. The system's instantaneous density operator $\boldsymbol{\rho}(t)$ can be decomposed in this basis.

An explicit description of decoherence hinges on a key unknown: the set of pointer states that are selected by the environment. Physically, these should correspond to the most stable, quasi-classical descriptions of the particle's trajectory. Consequently, the search naturally focuses on the momentum eigenstates $|p\rangle$, the position eigenstates $|x\rangle$, (as suggested by the decoherence mechanisms described in equations (16) and (17)), or hybrid states, such as coherent states, that combine both position and momentum localisation [39]–[43]. Indeed, the interaction Hamiltonian describing the coupling between a system and its environment commonly involves the position and/or momentum operators, making these observables natural candidates for environment-induced superselection [3], [17], [21]. In this sense, the environment acts as a continuous monitoring device through position-sensitive interactions and momentum-dependent scattering processes. Crucially, any rigorous selection or identification of pointer states must incorporate the constraints of the uncertainty principle. A rigorous framework for selecting pointer states must explicitly respect the quantum uncertainty relation, which is

$$\left(\sigma_{pp}\right)\left(\sigma_{xx}\right) - \left(\sigma_{px}\right)^2 \geq \frac{(\hbar)^2}{4} \tag{18}$$

where σ_{pp}, σ_{xx} and σ_{px} are respectively variances and covariance of momentum and coordinate. Selecting a pure momentum eigenstate $|p\rangle$ implies $\sigma_{pp} = 0$, which is unphysical as it requires $\sigma_{xx} = \infty$. Selecting a pure position eigenstate $|x\rangle$ implies $\sigma_{xx} = 0$, which is equally unphysical as it requires $\sigma_{pp} = \infty$. If we rigorously take this conclusion into account, it is clear that even the standard exponential decay expressions (16) and (17) are ill defined (correspond to idealisations that neglect the fundamental minimum uncertainty of quantum states).

Therefore, viable pointer states must be states that optimally localise both position and momentum within the bounds of the uncertainty principle. Furthermore, in classical physics, the one dimensional motion of a particle is fully described by its instantaneous position $x(t)$ and momentum $p(t)$. The set of all possible simultaneous pairs (p, x) defines the particle's classical phase space, and each such pair represents an elementary basis state of motion. It follows that the quantum pointer states $|\alpha\rangle$ should be those that most closely approximate these classical basis states (p, x). According to the uncertainty relation (18), the existence of a state with exact simultaneous determination of p and x ($\sigma_{xx} = 0$ and $\sigma_{pp} = 0$) is impossible in quantum physics. The quantum states that most closely resemble the classical phase space point (p, x) are not eigenstates of position or momentum alone, but rather the states denoted $|\langle z\rangle\rangle$ which saturate the uncertainty relation or more generally the states denoted $|n, \langle z\rangle\rangle$ [4], [36]–[38], [69]. These latter states are closely related to the states $|\langle z\rangle\rangle$ and can be regarded as their generalisation [36].

4.2 Joint Momentum-Coordinate States and Quantum Phase Space

The minimum-uncertainty states $|\langle z\rangle\rangle$, called joint momentum coordinate states form the cornerstone for defining the QPS, as they provide the closest possible quantum analogue to a classical point (p, x). The coordinate wavefunction corresponding to these states are given by the following relation [4], [36]:

$$\langle x|\langle z\rangle\rangle = \frac{1}{(2\pi\mathcal{X})^{1/4}} e^{-\frac{\mathcal{B}}{\hbar^2}(x-\langle x\rangle)^2+\frac{i}{\hbar}\langle p\rangle x} \tag{19}$$

in which:

- $\langle p\rangle$ and $\langle x\rangle$ are the expectation values of momentum and coordinate operators $\boldsymbol{p}$ and $\boldsymbol{x}$ corresponding to a state $|\langle z\rangle\rangle$ itself, respectively:
$$\langle x\rangle = \langle\langle z|\boldsymbol{x}|\langle z\rangle\rangle \quad and \quad \langle p\rangle = \langle\langle z|\boldsymbol{p}|\langle z\rangle\rangle \tag{20}$$

- B is a parameter which is related to the variance and covariance, $\sigma_{pp} = P$, $\sigma_{xx} = X$ and $\sigma_{px} = Q$ of momentum and coordinate operators, $\boldsymbol{p}$ and $\boldsymbol{x}$ corresponding to the states $|\langle z\rangle\rangle$:
$$\mathcal{B} = \frac{\hbar^2}{4\mathcal{X}}\left[1-\frac{2i}{\hbar}\mathcal{Q}\right] = \frac{\mathcal{P}}{1+4\frac{(\mathcal{Q})^2}{\hbar^2}}\left[1-\frac{2i}{\hbar}\mathcal{Q}\right] \tag{21}$$

$$\begin{cases} \mathcal{P} = \langle\langle z|(\boldsymbol{p}-\langle p\rangle)^2|\langle z\rangle\rangle = \langle\langle z|(\boldsymbol{p})^2|\langle z\rangle\rangle - (\langle p\rangle)^2 \\ \mathcal{X} = \langle\langle z|(\boldsymbol{x}-\langle x\rangle)^2|\langle z\rangle\rangle = \langle\langle z|(\boldsymbol{x})^2|\langle z\rangle\rangle - (\langle x\rangle)^2 \\ \mathcal{Q} = \frac{1}{2}\langle\langle z|(\boldsymbol{p}-\langle p\rangle)(\boldsymbol{x}-\langle x\rangle)+(\boldsymbol{x}-\langle x\rangle)(\boldsymbol{p}-\langle p\rangle)|\langle z\rangle\rangle = \frac{1}{2}\langle\langle z|\boldsymbol{px}+\boldsymbol{xp}|\langle z\rangle\rangle - \langle p\rangle\langle x\rangle \end{cases} \tag{22}$$

P, X and Q correspond to the saturation of the uncertainty relation (18),

$$det[\mathcal{G}] = \mathcal{P}\mathcal{X} - (\mathcal{Q})^2 = \frac{(\hbar)^2}{4} \qquad where \qquad \mathcal{G} = \begin{pmatrix} \mathcal{P} & \mathcal{Q} \\ \mathcal{Q} & \mathcal{X} \end{pmatrix} \tag{23}$$

G is the momentum coordinate variances covariance matrix corresponding to the state $|\langle z\rangle\rangle$. It can be shown that the states $|\langle z\rangle\rangle$ are eigenstates of the operator $\boldsymbol{z} = \boldsymbol{p} - \frac{2i}{\hbar}\mathcal{B}\,\boldsymbol{x}$. The corresponding eigenvalue equation is

$$\boldsymbol{z}|\langle z\rangle\rangle = \left[\langle p\rangle - \frac{2i}{\hbar}\mathcal{B}\langle x\rangle\right]|\langle z\rangle\rangle = \langle z\rangle|\langle z\rangle\rangle \tag{24}$$

and justifies the notation $|\langle z\rangle\rangle$.

The QPS is rigorously defined as the set $\{(\langle p\rangle, \langle x\rangle)\}$ of expectation-value pairs $(\langle p\rangle, \langle x\rangle)$ for given values of the variances and covariance P, X and Q satisfying (23) [4], [36].

This is not classical phase space: each QPS point or centroid ($\langle p\rangle$, $\langle x\rangle$) carries an intrinsic minimum quantum uncertainty encoded in the covariance matrix G. The states $|\langle z\rangle\rangle$ are the most general minimum-uncertainty states. They allow arbitrary variances P, X and covariance Q, corresponding to arbitrary squeezing and rotation of the uncertainty ellipse in phase space. They are therefore analogous to what are called squeezed states in the literature [39]–[43]. The set $\{|\langle z\rangle\rangle\}$ is then an overcomplete frame that provides a resolution of the identity over the QPS [4], [36]

$$\iint |\langle z\rangle\rangle\langle\langle z\rangle| \frac{d\langle p\rangle d\langle x\rangle}{h} = \mathbb{I} \tag{25a}$$

Any arbitrary instantaneous motional state $\boldsymbol{\rho}(t)$ of the particle can then be expressed in the overcomplete frame $\{|\langle z\rangle\rangle\}$

$$\begin{cases} \boldsymbol{\rho}(t) = \iint \iint \rho(\langle z\rangle, \langle z\rangle', t)|\langle z\rangle\rangle\langle\langle z\rangle'| \frac{d\langle p\rangle d\langle x\rangle}{h} \frac{d\langle p\rangle' d\langle x\rangle'}{h} \\ \rho(\langle z\rangle, \langle z\rangle', t) = \langle z(t)|\boldsymbol{\rho}(t)|z'(t)\rangle \end{cases} \tag{25b}$$

Here, h is the Planck constant.

4.3 Time-Dependent Orthonormal Basis $\{|n, \langle z(t)\rangle\rangle\}$ Associated to the QPS

The overcomplete frame $\{|\langle z\rangle\rangle\}$ is non-orthogonal: $\langle\langle z\rangle|\langle z\rangle'\rangle \neq 0$. Therefore, strict decoherence (the exact vanishing of off-diagonal density matrix elements) cannot be fully described in this frame. In fact, as discussed in Section 2, rigorous decoherence requires the pointer states to be mutually orthogonal so that the reduced density matrix becomes strictly diagonal. Moreover, the stronger objectivity criterion of SBS explicitly demands such orthogonality to guarantee that different environmental fragments contain perfectly distinguishable information about the system [27], [28]. Since the overcomplete frame $\{|\langle z\rangle\rangle\}$ fails this fundamental orthogonality requirement, the construction of a second, orthonormal basis should be considered.

To obtain an adequate orthonormal basis, we consider a time-dependent centroid $\langle z\rangle = \langle z(t)\rangle$ and construct an orthonormal Fock-like basis $\{|n, \langle z(t)\rangle\rangle\}$ above it by using ladder operators. The relations between the states $|\langle z(t)\rangle\rangle$ and $|n, \langle z(t)\rangle\rangle$ are analogous to the relations between the ground state of a harmonic oscillator and its excited states. To formally express this relations, we introduce ladder operators $\boldsymbol{Z}$ and $\boldsymbol{Z}^\dagger$ that satisfy the following relation [36]:

$$\begin{cases} \boldsymbol{Z} = i\frac{\sqrt{\mathcal{X}}}{\hbar}(\boldsymbol{z} - \langle z\rangle) = \frac{2\mathcal{B}}{\hbar^2}\sqrt{\mathcal{X}}\left[(\boldsymbol{x} - \langle x\rangle) + \frac{i\hbar}{2\mathcal{B}}(\boldsymbol{p} - \langle p\rangle)\right] \\ \boldsymbol{Z}^\dagger = -i\frac{\sqrt{\mathcal{X}}}{\hbar}(\boldsymbol{z}^\dagger - \langle z\rangle^*) = \frac{2\mathcal{B}}{\hbar^2}\sqrt{\mathcal{X}}\left[(\boldsymbol{x} - \langle x\rangle) - \frac{i\hbar}{2\mathcal{B}}(\boldsymbol{p} - \langle p\rangle)\right] \\ \left[\boldsymbol{Z}, \boldsymbol{Z}^\dagger\right] = \boldsymbol{Z}\boldsymbol{Z}^\dagger - \boldsymbol{Z}\boldsymbol{Z}^\dagger = 1 \end{cases} \tag{26}$$

We can then consider the following relations, which define the excited states $|n, \langle z(t)\rangle\rangle$:

$$\begin{cases} |n,\langle z\rangle\rangle=\frac{(\boldsymbol{Z}^\dagger)^n}{\sqrt{n!}}|\langle z\rangle\rangle \\ \boldsymbol{Z}|n,\langle z\rangle\rangle=\sqrt{n}|n-1,\langle z\rangle\rangle \\ \boldsymbol{Z}^\dagger|n,\langle z\rangle\rangle = \sqrt{n+1}|n+1,\langle z\rangle\rangle \\ \aleph|n,\langle z\rangle\rangle = \boldsymbol{Z}^\dagger\boldsymbol{Z}|n,\langle z\rangle\rangle = n|\langle z\rangle\rangle \end{cases} \tag{27}$$

In relation (22), n is a positive integer index which labels the excitation level above the reference state $|\langle z\rangle\rangle$. A state $|n, \langle z\rangle\rangle$ is an eigenstate of the number operator $\aleph = \boldsymbol{Z}^\dagger\boldsymbol{Z}$ constructed from the ladder operators $\boldsymbol{Z}$ and $\boldsymbol{Z}^\dagger$. The corresponding eigenvalue is n. For a fixed $\langle z\rangle = \langle z(t)\rangle$ (at any fixed instant t), the set $\{|n, \langle z\rangle\rangle\}$ form a complete orthonormal basis:

$$\begin{cases} \sum_{n=0}^{+\infty}\big||n, \langle z\rangle\rangle\,\langle n, \langle z\rangle| = \mathbb{I} \\ \langle n, \langle z\rangle|m, \langle z\rangle\rangle = \delta_{nm} \end{cases} \tag{28}$$

The time-dependent orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ encodes the particle's mean trajectory in QPS, with the centroid $\langle z(t)\rangle = \langle p(t)\rangle - \frac{2i}{\hbar}\mathcal{B}(t)\langle x(t)\rangle$ defining this path. This mean trajectory can be considered the quantum analogue of a classical trajectory.

Any arbitrary instantaneous motional state $\boldsymbol{\rho}(t)$ of the particle can be expressed in the time-dependent orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ as

$$\begin{cases} \boldsymbol{\rho}(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\rho_m^n(t)\,|n, \langle z(t)\rangle\rangle\,\langle m, \langle z'(t)\rangle| \\ \rho_m^n(t) = \langle n, z(t)|\boldsymbol{\rho}(t)|m, z(t)\rangle \end{cases} \tag{29}$$

4.4 Identification of Pointer States Within the QPS Framework, Role of the Hamiltonian and Distinction Between Decoherence and Thermalisation

A potential ambiguity arises in the identification of pointer states: are the pointer states the continuous squeezed states $|\langle z\rangle\rangle$ or the time-dependent orthonormal Fock-like states $|n, \langle z(t)\rangle\rangle$? The QPS formalism resolves this by assigning distinct complementary roles to each, and by showing that all of the main pointer state criteria outlined in Section 2 converge in this unified structure. We can consider the following two levels:

- The overcomplete continuous frame $\{|\langle z\rangle\rangle\}$ defines the geometric framework of the QPS. Each state $|\langle z\rangle\rangle$ corresponds to a localised wave packet centred at ($\langle p\rangle$, $\langle x\rangle$) and saturates the uncertainty relation. This framework provides the underlying QPS corresponding to the manifold of possible classical-like trajectories, ie mean trajectories. However, the non-orthogonality of the states $|\langle z\rangle\rangle$ prevents strict diagonalisation of the density matrix.

- The time-dependent orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$ can serve as the rigorous pointer basis for strict decoherence, enabling diagonalisation, although the excited states $|n, \langle z(t)\rangle\rangle$ do not saturate uncertainty.

All criteria from Section 2.2 could converge within this framework: einselection, Quantum Darwinism, and SBS identify the full orthonormal basis $\{|n, \langle z(t)\rangle\rangle\}$. The predictability sieve or minimal entropy production single out the ground state $|\langle z\rangle\rangle$ as the most classically predictable pure state.

Although the previous discussion shed light on the identification of pointer states within the QPS framework, the distinction between the two quantum numbers $\langle z\rangle$ and n and their respective roles in decoherence and thermalisation can only be fully understood by examining the structure of the total Hamiltonian. Consider the general system-bath Hamiltonian

$$\boldsymbol{H_{SB} = H_S + H_B + H_I = H + H_B + H_I} \tag{30}$$

where $\boldsymbol{H_S = H}$ is the system self-Hamiltonian, $\boldsymbol{H_B}$ the bath (environment) Hamiltonian and $\boldsymbol{H_I}$ is the interaction Hamiltonian. A general form of $\boldsymbol{H_I}$ is given by the relation (1). Within the QPS framework, we assume that the environment monitors position, momentum, or both, so the coupling operators $\boldsymbol{A_k}$ are typically linear combinations of $\boldsymbol{p}$ and $\boldsymbol{x}$

$$\boldsymbol{A_k} = \alpha_k \boldsymbol{p} + \beta_k \boldsymbol{x} \tag{31}$$

As highlighted by relation (2), the pointer states are the approximate eigenstates of these operators $\boldsymbol{A_k}$, which correspond to the states $|\langle z\rangle\rangle$ and $|n, \langle z(t)\rangle\rangle$ as may be expected. It corresponds to the fact that environment therefore continuously measures the centroid ($\langle p\rangle$, $\langle x\rangle$). Decoherence then destroys coherences between different $\langle z\rangle$, which correspond to different classical-like trajectories. The system self-Hamiltonian governs the evolution related to the excitation number n. The key question is whether $\boldsymbol{H_S = H}$ commutes with the pointer observables $\boldsymbol{A_k}$:

- if $[\boldsymbol{H}, \boldsymbol{A_k}] = 0$, the pointer basis $\{|n, \langle z(t)\rangle\rangle\}$ coincides with the energy eigenbasis. In this case, the index n also labels energy eigenstates and decoherence destroys coherences between different n on the fast timescale τ_D. Subsequently, thermalisation drives the diagonal populations ρ_n^n towards the Gibbs distribution on the slow timescale τ_R, or
- if $[\boldsymbol{H}, \boldsymbol{A_k}] \neq 0$, the pointer basis $\{|n, \langle z(t)\rangle\rangle\}$ differs from the energy eigenbasis. The environment measures the centroid $\langle z\rangle$ so decoherence destroys coherences between different centroids. The system self-Hamiltonian couples different n levels and the environment's influence (through the interaction) acts on all coherences in the pointer basis, including those between different n for the same

$\langle z \rangle$, but the rate may depend on n. Thermalisation acts on the diagonal populations ρ_n^n, driving them towards the Gibbs distribution.

In both cases, the separation of timescales $\tau_D \ll \tau_R$ arises because decoherence (driven by diffusion) is typically much faster than thermalisation (driven by dissipation). The QPS framework therefore provides a first principles geometric distinction between the two processes.

4.5 Relations Between Environment Properties, Decoherence Regime and the Structure of the QPS

Based on the results of the previous section, there is a direct correspondence between the set of pointer states for a particle's motion and its QPS. Consequently, the characteristics of the environment, which are reflected in the nature of the pointer states, are directly linked to the structure of the particle's QPS. This structure is defined by the variances covariance matrix $\mathcal{G} = \begin{pmatrix} \mathcal{P} & \mathcal{Q} \\ \mathcal{Q} & \mathcal{X} \end{pmatrix}$ of the "ground states" $|\langle z \rangle\rangle$ defining the QPS. It follows that the properties of the environment should also be described with these quantities. Two main cases can be distinguished: stationary and reactive environments.

Stationary Environment (Markovian Regime)

This case occurs when the environment's intrinsic characteristics, such as its spectral density, temperature, and correlation functions, are not significantly perturbed or reconfigured by its interaction with the particle. Conceptually, the environment is assumed to be so large or have such fast internal dynamics that the energy and information exchanged with the particle cause negligible back action. While the environment inevitably captures information about the particle's state (for example, through scattering or field emissions), this transfer does not substantially alter the environment's global statistical properties or its future dynamical rules. Consequently, the environment retains no lasting "memory" of any specific interaction event; it resets rapidly to its equilibrium state, behaving as a memoryless Markovian reservoir. This type of environment corresponds precisely to the Markovian limit of decoherence. In this regime, the system's dynamics are local in time, described by master equations with constant damping rates, such as the Lindblad form. This theoretical description is physically justified when the environment's correlation time is vanishingly short compared to the system's dynamical timescale. The mathematical signature of this limit within our QPS framework is that the corresponding variance–covariance matrix for the environmental noise is time-independent, reflecting stationary statistical properties.

$$\dot{\mathcal{X}} = \frac{d\mathcal{X}}{dt} = 0 \quad \dot{\mathcal{P}} = \frac{d\mathcal{X}}{dt} = 0 \quad \dot{\mathcal{Q}} = \frac{d\mathcal{Q}}{dt} = 0 \quad \dot{\mathcal{G}} = \frac{d\mathcal{G}}{dt} = \begin{pmatrix} \dot{\mathcal{P}} & \dot{\mathcal{Q}} \\ \dot{\mathcal{Q}} & \dot{\mathcal{X}} \end{pmatrix} = 0 \tag{32}$$

It is this constancy in the second order moments that formally justifies the designation "stationary" for the environment in this model. This stationarity ensures that the

decoherence channels and diffusion processes they induce are uniform over time, leading to predictable exponential decay of coherences.

Reactive Environment (Non-Markovian Regime)

This regime describes an environment whose intrinsic physical and statistical properties, such as its effective spectral density, temperature distribution and, most crucially, its temporal correlation functions, are notably perturbed and reshaped by the particle's presence and dynamics. Unlike a passive Markovian bath that remains inert and unchanging, a reactive environment dynamically adapts to the ongoing interaction. It does not simply act as a sink for information; rather, the absorption of information about the particle's state (for example, its position, momentum or internal energy) induces a structural or dynamical change in the environment itself. This exchange creates a bidirectional feedback loop: the system influences the environment, and the environment's modified state, in turn, exerts a new influence back on the system at a later time. This is the essence of a memory effect; the environment effectively retains a non-trivial dynamical trace of past interactions, which actively influences the future evolution of the system's quantum coherences. Consequently, the decoherence process becomes history dependent. Mathematically, this complex temporally non-local behaviour is captured by a time-dependent variance–covariance matrix

$$\dot{\mathcal{X}} = \frac{d\mathcal{X}}{dt} \neq 0 \qquad \dot{\mathcal{P}} = \frac{d\mathcal{X}}{dt} \neq 0 \qquad \dot{\mathcal{Q}} = \frac{d\mathcal{Q}}{dt} \neq 0 \qquad \dot{\mathcal{G}} = \frac{d\mathcal{G}}{dt} = \begin{pmatrix} \dot{\mathcal{P}} & \dot{\mathcal{Q}} \\ \dot{\mathcal{Q}} & \dot{\mathcal{X}} \end{pmatrix} \neq 0 \tag{33}$$

This temporal dependence signifies that the statistical moments (the "noise strength" and inter-mode correlations) governing the environmental fluctuations are not stationary but evolve in response to the system's history, providing a direct quantitative signature of the environment's reactive and memory bearing nature. The "breathing" or rotation of the uncertainty ellipse in phase space provides a direct geometric representation of non-exponential decoherence, recoherence and information backflow, corresponding precisely to the non-Markovian regime.

5 Illustrative Application: Extending the Hu–Paz–Zhang Model

This illustrative application demonstrates how the general QPS framework connects environmental parameters to QPS structure, naturally accommodating and generalising the standard HPZ model of quantum Brownian motion.

5.1 The Standard Hu–Paz–Zhang Model

The HPZ master equation is a non-Markovian quantum master equation that describes the time evolution of a quantum harmonic oscillator coupled to a general bosonic bath environment (heat bath). First derived by Hu *et al.* [44] in 1992 using the Feynman-Vernon path-integral influence functional method [70], it serves as a foundational pillar

in the study of quantum Brownian motion, decoherence, and open quantum systems [9], [30], [44]–[47], [71]–[73].

System-Bath Hamiltonian

The model considers a harmonic oscillator of mass M_0 and frequency Ω_0 linearly coupled to a bath of harmonic oscillators with masses m_k and frequency ω_k that constitutes the environment. The system-bath Hamiltonian is

$$\boldsymbol{H_{SB}} = \boldsymbol{H_S} + \boldsymbol{H_B} + \boldsymbol{H_I} \quad \text{with} \quad \begin{cases} \boldsymbol{H_S} = \frac{(\boldsymbol{p})^2}{2M} + \frac{1}{2}M(\Omega)^2(\boldsymbol{x})^2 \\ \boldsymbol{H_B} = \sum_{\boldsymbol{k}} \frac{(\boldsymbol{p_k})^2}{2m_k} + \frac{1}{2}m_k(\omega_k)^2(\boldsymbol{q}_k)^2 \\ \boldsymbol{H_I} = \boldsymbol{x}\sum_{\boldsymbol{k}} C_k \boldsymbol{q}_k \end{cases} \tag{34}$$

The bath spectral density corresponding to the coupling with the system is

$$J(\omega) = \frac{\pi}{2}\sum_k \frac{(C_k)^2}{m_k\omega_k}\delta(\omega - \omega_k) \tag{35}$$

Since realistic environments contain a very large number of degrees of freedom, one usually adopts the continuum limit in which the discrete set of bath oscillators is replaced by a continuous distribution of frequencies. Accordingly, the discrete spectral density becomes

$$J(\omega) = \frac{\pi}{2}g(\omega)\frac{[C(\omega)]^2}{m(\omega)\omega} \tag{36}$$

where $g(\omega)$ is the density of bath modes and $C(\omega)$ and $m(\omega)$ denote the frequency-dependent oscillator masses and coupling strengths, respectively.

In the standard HPZ model, only the bath coordinates $\boldsymbol{q_k}$ couple with the system coordinate $\boldsymbol{x}$; there is no coupling between the bath momenta $\boldsymbol{p_k}$ with the system momentum $\boldsymbol{p}$. This asymmetry is the defining feature of the standard model and can serve as the starting point for a generalisation within the QPS formalism.

Master Equation

The state of the combined system is described by a density operator $\boldsymbol{\rho}_{SB}$, evolving via the Liouville–Von Neumann equation. Tracing over the bath degrees of freedom yields the reduced density operator (the system density operator) $\boldsymbol{\rho_S} = \boldsymbol{\rho}$ which obeys the exact HPZ master equation for quantum Brownian motion in a general Gaussian environment [9], [30]

$$\frac{d\boldsymbol{\rho}}{dt} = -\frac{i}{\hbar}[\,\boldsymbol{H},\boldsymbol{\rho}] - \frac{i}{\hbar}\Gamma(t)[\boldsymbol{x},\{\boldsymbol{p},\boldsymbol{\rho}\}] - \frac{D_{pp}(t)}{(\hbar)^2}\big[\boldsymbol{x},[\boldsymbol{x},\boldsymbol{\rho}]\big] + \frac{D_{xp}(t)}{(\hbar)^2}[\boldsymbol{x},[\boldsymbol{p},\boldsymbol{\rho}]] \tag{37}$$

where $\boldsymbol{H}$ is the renormalised system Hamiltonian

$$\boldsymbol{H} = \boldsymbol{H_S} + \frac{1}{2}M\delta\Omega^2(t)(\boldsymbol{x})^2 \tag{38}$$

$\delta\Omega^2(t)$ correspond to a transient time-dependent frequency renormalisation: time-dependent shift in the effective oscillation frequency of the Brownian particle, induced by the bath's back-reaction. Physically, the particle "drags" a cloud of bath excitations with it, modifying its effective inertia or potential.

$\Gamma(t)$, $D_{pp}(t)$ and $D_{xp}(t)$ are respectively the friction/dissipation coefficient (driving thermalisation), the normal diffusion coefficient (driving decoherence), and the anomalous diffusion coefficient (coupling position and momentum, modifying squeezing).They are determined by the bath spectral density $J(\omega)$ and the bath temperature T [9], [30], [44],[46],[71]-[72].

A widely used example is the Lorentz–Drude spectral density, which models an ohmic environment at low frequencies together with a Lorentzian high-frequency cutoff [9], [73]

$$J(\omega) = M\gamma_0\omega\frac{(\omega_c)^2}{(\omega_c)^2+(\omega)^2} \tag{39}$$

where γ_0 is the damping rate and ω_c the cutoff frequency. It can be shown in particular that in the Markovian limit (ω_c→∞), the coefficients reduce to constant (time-independent) values (k_B is the Boltzmann constant):

$$\Gamma(t) \to \gamma_0 \quad D_{pp}(t) \to 2M\gamma_0 k_B T \quad D_{xp}(t) \to 0 \tag{40}$$

Thus the HPZ master equation reduces to the (Markovian) standard high-temperature Caldeira-Leggett master equation [9], [74]

$$\frac{d\boldsymbol{\rho}}{dt} = -\frac{i}{\hbar}[\,\boldsymbol{H},\boldsymbol{\rho}] - \frac{i}{\hbar}\gamma_0[\boldsymbol{x},\{\boldsymbol{p},\boldsymbol{\rho}\}] - \frac{2M\gamma_0 k_B T}{(\hbar)^2}[\boldsymbol{x},[\boldsymbol{x},\boldsymbol{\rho}]] \tag{41}$$

Beyond the Lorentz–Drude model, different choices of spectral density allow one to describe different environmental properties, including ohmic, sub-ohmic, super-ohmic, and structured (coloured) reservoirs, leading to different memory and dissipation effects [9], [30], [73].

5.2 Extension of the Hu–Paz–Zhang Model Within the Quantum Phase Space Formalism

The standard HPZ master equation is fully consistent with the QPS formalism. It can provide explicit relations between the variance–covariance matrix $G(t)$ of the QPS ground states and the HPZ coefficients, and it can be formulated in both the overcomplete frame $\{|\langle z\rangle\rangle\}$ and the orthonormal pointer basis $\{|n, \langle z(t)\rangle\rangle\}$. However, while the standard HPZ model is sufficient for environments that monitor only the system's position, the QPS formalism treats position and momentum as complementary phase-space variables. It is therefore more natural and particularly relevant for nanoscience to consider the extended version of the HPZ model that includes momentum coupling.

Extended System-Bath Hamiltonian

The standard HPZ Hamiltonian couples the bath only to the particle's position $\boldsymbol{x}$ as shown in the relation (34). To describe environments that monitor both position and momentum, a situation common in nanoscale devices where multiple decoherence channels are active, the interaction Hamiltonian must be modified to include momentum coupling. The inclusion of momentum-dependent couplings in quantum Brownian motion has been explored in recent studies, where the standard HPZ equation emerges as a special case [75]–[77]. The present work differs from these prior generalisations in both motivation and scope. While previous studies focused on deriving generalised master equations, our approach is rooted in the QPS formalism and aims to illustrate its power by explicitly linking environmental parameters, encoded in the spectral-density matrix, to the QPS geometry and the emergence of classical-like trajectories. Furthermore, the QPS formalism naturally accommodates both Markovian and non-Markovian regimes through the time-dependence of the covariance matrix $G(t)$, providing a unified geometric language that is absent in previous momentum-coupling generalisations. The most general interaction Hamiltonian including this coupling and which is naturally compatible with the QPS framework (cf. equation (31)) is

$$\boldsymbol{H_I} = \boldsymbol{p}\sum_{\boldsymbol{k}}(A_k\boldsymbol{q}_k + B_k\boldsymbol{p}_{\boldsymbol{k}}) + \boldsymbol{x}\sum_{\boldsymbol{k}}(C_k\boldsymbol{q}_{\boldsymbol{k}} + D_k\boldsymbol{p}_{\boldsymbol{k}}) \tag{42}$$

where the system momentum $\boldsymbol{p}$ and position $\boldsymbol{x}$ are allowed to couple linearly to both the bath coordinates $\boldsymbol{q}_{\boldsymbol{k}}$ and bath momenta $\boldsymbol{p}_{\boldsymbol{k}}$. This general interaction encompasses the standard HPZ model as a particular case ($\boldsymbol{A}_{\boldsymbol{k}} = 0$, $B_k = 0$, $D_k = 0$) while also describing momentum-mediated and mixed coupling mechanisms that may arise in quantum transport, kinetic interactions, or engineered quantum reservoirs.

Unlike the standard HPZ model, which is completely characterised by a single spectral density (equation (35)), the generalised interaction requires a spectral-density matrix

$$J(\omega) = \begin{pmatrix} J_{pp}(\omega) & J_{px}(\omega) \\ J_{xp}(\omega) & J_{xx}(\omega) \end{pmatrix} \tag{43}$$

whose elements describe the correlations associated with the different coupling channels. For the discrete bath, the spectral densities are defined by

$$\begin{cases} J_{pp}(\omega) = \frac{\pi}{2}\sum_k [\frac{(A_k)^2}{m_k\omega_k} + m_k\omega_k(B_k)^2]\delta(\omega-\omega_k) \\ J_{xx}(\omega) = \frac{\pi}{2}\sum_k [\frac{(C_k)^2}{m_k\omega_k} + m_k\omega_k(D_k)^2]\delta(\omega-\omega_k) \\ J_{px}(\omega) = \frac{\pi}{2}\sum_k [\frac{A_kC_k}{m_k\omega_k} + m_k\omega_kB_kD_k]\delta(\omega-\omega_k) \\ J_{xp}(\omega) = J_{px}(\omega) \end{cases} \tag{44}$$

As in the standard HPZ model, the bath is assumed to contain a macroscopic number of oscillators. In the continuum (thermodynamic) limit, the discrete frequency spectrum is replaced by a continuous distribution of modes, and the spectral densities become smooth functions of the frequency,

$$\begin{cases} J_{pp}(\omega) = \frac{\pi}{2}g(\omega)\left[\frac{[A(\omega)]^2}{m(\omega)\omega} + m(\omega)\omega[B(\omega)]^2\right] \\ J_{xx}(\omega) = \frac{\pi}{2}g(\omega)\left[\frac{[C(\omega)]^2}{m(\omega)\omega} + m(\omega)\omega[D(\omega)]^2\right] \\ J_{px}(\omega) = \frac{\pi}{2}g(\omega)\left[\frac{A(\omega)C(\omega)}{m(\omega)\omega} + m(\omega)\omega B(\omega)D(\omega)\right] \\ J_{xp}(\omega) = J_{px}(\omega) \end{cases} \tag{45}$$

where $g(\omega)$ is the density of bath modes.

Generalised Master Equation

By performing a partial trace over the environmental degrees of freedom, we obtain the system's density operator, $\boldsymbol{\rho}$, which satisfies the following generalised non-Markovian master equation

$$\frac{d\boldsymbol{\rho}}{dt} = -\frac{i}{\hbar}[\,\boldsymbol{H},\boldsymbol{\rho}] - \frac{i}{\hbar}\sum_{i,j=p,x}\Gamma_{ij}(t)\left[\boldsymbol{q}_i,\{\boldsymbol{q}_j,\boldsymbol{\rho}\}\right] + \frac{1}{(\hbar)^2}\sum_{i,j=p,x}D_{ij}(t)\left[\boldsymbol{q}_i,[\boldsymbol{q}_j,\boldsymbol{\rho}]\right] \tag{46}$$

where

- $\boldsymbol{q}_p = \boldsymbol{p}$ and $\boldsymbol{q}_x = \boldsymbol{x}$
- $\boldsymbol{H}$ is the renormalised system Hamiltonian

$$\boldsymbol{H} = \boldsymbol{H_S} + \frac{1}{2}\delta\Omega^2(t)(\boldsymbol{x})^2 - \frac{\delta M(t)}{2M}(\boldsymbol{p})^2 + \frac{\delta C(t)}{2}(\boldsymbol{xp} + \boldsymbol{px}) \tag{47}$$

$\delta\Omega^2(t), \delta M(t)$ and $C(t)$ denote the time-dependent renormalisations of the oscillator frequency, effective mass, and position-momentum coupling induced by the interaction with the environment.

The coefficients $\Gamma_{ij}(t)$ constitute the dissipation matrix, whereas $D_{ij}(t)$ form the diffusion matrix. Their diagonal elements describe the dissipation and diffusion associated with the position and momentum coupling channels, whereas the off-diagonal elements account for the correlations induced by the mixed system-bath interactions. All these coefficients are uniquely determined by the bath temperature T and the spectral-density matrix $J_{ij}(\omega)$ through the corresponding dissipation and noise kernels.

Expectation Values and Variance–Covariance

As we have for the expectation values and variance–covariance of position and momentum the following relations

$$\begin{cases} \langle p\rangle = Tr(\boldsymbol{\rho p}) \\ \langle x\rangle = Tr(\boldsymbol{\rho x}) \\ \mathcal{P} = Tr[\boldsymbol{\rho}(\boldsymbol{p} - \langle p\rangle)^2] \\ \mathcal{X} = Tr[\boldsymbol{\rho}(\boldsymbol{x} - \langle x\rangle)^2] \\ \mathcal{Q} = \frac{1}{2}Tr[\boldsymbol{\rho}(\boldsymbol{px} + \boldsymbol{xp})] - \langle p\rangle\langle x\rangle \end{cases} \tag{48}$$

It can be deduced from the general master equation (46) that we have the following relation for the evolution of the centroid ($\langle p(t)\rangle$, $\langle x(t)\rangle$) corresponding to the states $|n, \langle z(t)\rangle\rangle$ and for the ground variance–covariance matrix $\mathcal{G} = \begin{pmatrix} \mathcal{P} & \mathcal{Q} \\ \mathcal{Q} & \mathcal{X} \end{pmatrix}$

$$\begin{cases} \frac{d\langle p\rangle}{dt} = -\left[\delta C(t) + 2\Gamma_{xp}(t)\right]\langle p\rangle - \left[M\left(\Omega^2 + \delta\Omega^2(t)\right) + 2\Gamma_{xx}(t)\right]\langle x\rangle \\ \frac{d\langle x\rangle}{dt} = \left[\frac{1-\delta M(t)}{M} - 2\Gamma_{pp}(t)\right]\langle p\rangle + \left[\delta C(t) - 2\Gamma_{px}(t)\right]\langle x\rangle \\ \frac{d\mathcal{P}}{dt} = -2\left[\delta C(t) + 2\Gamma_{xp}(t)\right]\mathcal{P} - 2[M(\Omega^2 + \delta\Omega^2(t)) + 2\Gamma_{xx}(t)]\mathcal{Q} + 2D_{pp} \\ \frac{d\mathcal{X}}{dt} = 2\left[\delta C(t) - 2\Gamma_{px}(t)\right]\mathcal{X} + 2\left[\frac{1-\delta M(t)}{M} - 2\Gamma_{pp}(t)\right]\mathcal{Q} + 2D_{xx} \\ \frac{d\mathcal{Q}}{dt} = \left[\frac{1-\delta M(t)}{M} - 2\Gamma_{pp}(t)\right]\mathcal{P} - [M(\Omega^2 + \delta\Omega^2(t)) + 2\Gamma_{xx}(t)]\mathcal{X} - 2\left[\Gamma_{px}(t) + \Gamma_{xp}(t)\right]\mathcal{Q} + 2D_{xp} \end{cases} \tag{49}$$

The equations in (49) correspond to the general non-Markovian regime (reactive environment) as described by the equation (33). These equations establish a direct link between the microscopic parameters of the extended HPZ model and the QPS geometry: the centroid dynamics describe classical-like trajectories influenced by the

environment's memory, while the variance–covariance equations govern the evolution and time-dependent deformation of the uncertainty ellipse. In the Markovian limit (stationary environment, equation (32)), the coefficient of the master equation become constant and we have the following reduction:

$$\Gamma_{ij}(t) \longrightarrow \Gamma_{ij}^{(0)} \quad \mathrm{D}_{ij}(t) \longrightarrow \mathrm{D}_{ij}^{(0)} \quad \delta\Omega^2(t) = 0 \quad \delta M(t) = 0 \quad \delta C(t) = 0 \tag{50}$$

The equation in (49) becomes

$$\begin{cases} \frac{d\langle p\rangle}{dt} = -2\Gamma_{xp}^{(0)}\langle p\rangle - \left(M\Omega^2 + 2\Gamma_{xx}^{(0)}\right)\langle x\rangle \\ \frac{d\langle x\rangle}{dt} = (\frac{1}{M} - 2\Gamma_{pp}^{(0)})\langle p\rangle - 2\Gamma_{px}^{(0)}\langle x\rangle \\ \Gamma_{xp}^{(0)}\mathcal{P} + (M\Omega^2 + 2\Gamma_{xx}^{(0)})\mathcal{Q} = D_{pp} \\ 2\Gamma_{px}^{(0)}\mathcal{X} - \left(\frac{1}{M} - 2\Gamma_{pp}^{(0)}\right)\mathcal{Q} = D_{xx} \\ -\left(\frac{1}{M} - 2\Gamma_{pp}^{(0)}\right)\mathcal{P} + \left(M\Omega^2 + 2\Gamma_{xx}^{(0)}\right)\mathcal{X} + 2\left[\Gamma_{px}^{(0)} + \Gamma_{xp}^{(0)}\right]\mathcal{Q} = -2D_{xp} \end{cases} \tag{51}$$

The equations in (51) correspond to the Markovian regime (stationary environment), where all coefficients have become time-independent and the transient renormalisations vanish. In this limit, the centroid dynamics reduce to a system of linear differential equations with constant coefficients, describing damped motion of the mean trajectory. The ground variance–covariance equations, however, become algebraic relations rather than differential equations, reflecting the stationarity of the environment. These algebraic relations define the fixed uncertainty ellipse in phase space: they explicitly link the constant diffusion and dissipation coefficients to the time-independent QPS geometry, formalising the concept that a stationary environment selects and "freezes" a specific set of quasi-classical trajectories. For the Markovian high temperature limit of the standard HPZ model: $\Gamma_{xx}^{(0)} = \Gamma_{pp}^{(0)} = \Gamma_{px}^{(0)} = 0, \Gamma_{xp}^{(0)} = \gamma_0$ and $D_{pp} = 0$, $D_{xx} = 0$, $D_{pp} = 2M\gamma_0 k_B T$

$$\begin{cases} \frac{d\langle p\rangle}{dt} = -2\gamma_0\langle p\rangle - M\Omega^2\langle x\rangle \\ \frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{M} \\ \mathcal{P} = Mk_BT \\ \mathcal{Q} = 0 \\ \mathcal{X} = \frac{\mathcal{P}}{M^2\Omega^2} = \frac{k_BT}{M\Omega^2} \end{cases} \tag{52}$$

According to equations in (52), the centroid dynamics reduce to classical-like damped oscillator equations. The equilibrium QPS geometry satisfies the equipartition theorem, with zero covariance ($Q = 0$) indicating an aligned non-squeezed uncertainty ellipse.

These results confirm that the QPS framework consistently reproduces the expected thermal equilibrium state of a harmonic oscillator.

Pointer States

The generalised interaction Hamiltonian (42) simultaneously couples both the position and momentum observables of the system to the environment. Since the QPS formalism is likewise constructed on the simultaneous treatment of coordinates and momenta, it is natural to expect that the pointer states are the states $|\langle z\rangle\rangle$ and $|n, \langle z(t)\rangle\rangle$ introduced within the QPS framework as discussed in Section 4.4. However, a rigorous demonstration of this conjecture would require:

- solving the generalised master equation (46) in the proposed QPS pointer-state basis;
- analysing the corresponding decoherence rates for the off-diagonal elements; and
- verifying explicitly that the pointer basis is indeed the proposed ones.

Such an analysis is beyond the scope of the present work because of the lengthy calculations involved, but it constitutes a promising direction for future research. The generalised HPZ master equation derived here provides the necessary dynamical framework for such an investigation. Beyond its theoretical interest, this generalised master equation may provide a suitable framework for modelling decoherence in nanoscale systems where both position and momentum interactions with the environment play a significant role. Potential applications include:

- Nanoelectromechanical resonators: Position- and momentum-dependent coupling to quantum point contacts and other mesoscopic detectors [49], [57], [62], [63], [78], [79].
- Optomechanical systems: Radiation-pressure interactions, photon recoil, and structured optical environments [50], [80], [81].
- Superconducting quantum circuits: Flux, charge, and dielectric noise [15], [77], [82], [83].
- Semiconductor quantum dots: Charge noise and electron–phonon coupling [52], [57], [84], [85].
- Hybrid quantum devices: Multiple decoherence channels and engineered environments [33], [34], [56], [68], [86], [87].

In these systems, non-Markovian effects and phase-space dynamics are known to influence coherence times and quantum control. In these systems, the QPS formalism may offer an alternative representation of decoherence dynamics based directly on the evolution of the phase-space moments, as given by equations (49).

The generalised master equation with its time-dependent coefficients naturally captures the non-Markovian features of these nanoscale environments, while the QPS geometry provides a clear physical interpretation of the decoherence dynamics and regimes with regard to the evolution of the uncertainty ellipse.

6 Discussion and Conclusion

This work has introduced and developed a novel theoretical framework grounded in the concept of QPS to address the fundamental and practical challenges of quantum decoherence in nanoscience. Our approach unifies the dual pillars of the decoherence process: the dynamical selection of pointer states and the characterisation of decoherence regimes. By connecting environmental properties to the geometry of QPS, we provide a powerful tool for modelling, analysing and potentially controlling quantum coherence at the nanoscale.

Our central achievement is the identification of potential pointer states for a particle's motional degrees of freedom as minimum-uncertainty states $|\langle z\rangle\rangle$ that saturate the uncertainty relation along with their "excited" counterparts $|n, \langle z(t)\rangle\rangle$. These states are directly related to the concept of QPS and resolve the theoretical impasse presented by idealised position or momentum eigenstates, which are physically inadmissible under the uncertainty principle. The states $|\langle z\rangle\rangle = |0, \langle z\rangle\rangle$ (the "ground states" defining the QPS) and their excitations $|n, \langle z(t)\rangle\rangle$ constitute the closest quantum analogue to a classical phase-space point. Consequently, they provide the natural, physically sound pointer basis into which a system (particle in motion) decoheres [1]–[6], [17]–[25].

The structure of the QPS is encoded in the variance-covariance matrix $\mathcal{G} = \begin{pmatrix} \mathcal{P} & \mathcal{Q} \\ \mathcal{Q} & \mathcal{X} \end{pmatrix}$ of these ground states $|\langle z\rangle\rangle$. We have demonstrated that this matrix is not static but is directly shaped by the environment, providing a unifying geometric criterion to distinguish decoherence regimes:

- Markovian (stationary) regime: This corresponds to a memoryless, broadband environment whose statistical properties are unchanging. Within our framework, this is characterised by a time-independent variance–covariance matrix ($\dot{\mathcal{G}} = 0$). In the Markovian limit of the extended HPZ model that we introduce in this article, we derive the algebraic relations (51), which explicitly link the constant diffusion and dissipation coefficients to the fixed uncertainty ellipse in phase space. This formalises the concept that a stationary environment selects and "freezes" a specific set of quasi-classical trajectories [6], [88].
- Non-Markovian (reactive) regime: This describes environments with memory, structure, or strong coupling, where past interactions influence future dynamics. Here, the environment is reactive, and its statistical imprint on the system evolves. Mathematically, this is captured by a time-dependent variance–

covariance matrix ($\dot{\mathcal{G}} \neq 0$). Using the generalised non-Markovian master equation derived from our extended HPZ model. It is shown how time-dependent diffusion and friction coefficients govern the evolution of $G(t)$ via the generalised equations (49). The resulting "breathing" or rotation of the uncertainty ellipse in phase space provides a direct geometric representation of non-exponential decoherence, recoherence and information backflow [29]–[35], [63]–[68], [76].

The extended interaction Hamiltonian (42), which gives rise to the spectral-density matrix (43) and the generalised HPZ master equation (46), accounts for all possible bilinear couplings between the system and the bath. From this generalised master equation, we derive explicit closed-form evolution equations for the centroid ($\langle p(t)\rangle$, $\langle x(t)\rangle$) of the classical-like trajectories and the ground variance–covariance matrix $G(t)$ directly linking the environmental parameters to the QPS geometry. This approach provides a suitable framework for modelling decoherence in nanoscale systems where both position and momentum interactions with the environment play a significant role, including nanoelectromechanical resonators [49], [57], [62], [63], [78]–[79], optomechanical systems [50], [80], [81], superconducting quantum circuits [15], [77], [82], [83], semiconductor quantum dots [52], [57], [84], [85] and hybrid quantum devices[33], [34], [56], [68], [86], [87].

A key strength of the QPS framework is its ability to seamlessly encompass both Markovian and non-Markovian dynamics within a single consistent mathematical language. It transcends the traditional modelling dichotomy by showing that both regimes are manifestations of how environmental properties, encoded in spectral densities and correlation functions, imprint themselves onto the QPS geometry. The QPS framework offers a clear and simple metric (the time dependence of $G(t)$) to quantify non-Markovian memory. This can guide the active harnessing of environmental back action for quantum advantage, such as using structured environments for enhanced quantum sensing or leveraging memory effects in reservoir engineering for information processing [9], [30], [31], [71], [72].

Beyond its implications for nanoscience and quantum technologies, the QPS framework developed here has profound significance for fundamental physics and the foundational problems of quantum mechanics. The identification of pointer states as the natural basis for decoherence addresses the quantum measurement problem by providing a rigorous, geometric mechanism for the emergence of classical behaviour from quantum dynamics [1]–[8] Moreover, the QPS formalism is not restricted to non-relativistic quantum mechanics. Recent developments have extended the concept of QPS to the relativistic domain, establishing a relativistic QPS that inherently incorporates both the uncertainty principle and relativistic covariance [36]–[38], [69], [89]–[91]. These developments suggest that the QPS framework may also provide a relativistic geometric framework for studying decoherence processes in contexts where relativistic effects become important. The relevance of such a relativistic study of decoherence spans nanoscale physics, fundamental physics, and cosmology. At the nanoscale, mechanical oscillators in quantum states provide sensitive probes of quantum gravitational effects, where

fluctuating spacetime induces momentum-basis decoherence [92], and gravitational time dilation itself can cause universal decoherence of composite particle superpositions [93].

Furthermore, open quantum dots in graphene offer a unique platform for studying relativistic pointer states, as graphene's Dirac fermions imitate massless relativistic particles [94]. At the most fundamental level, the Unruh effect plays a dual role as both a decoherence source and an enhancer of quantum correlations for accelerating quantum systems [95], while black holes and Killing horizons have been shown to decohere quantum superpositions through the emission of extremely low-frequency Hawking quanta [96]. In cosmology, decoherence is essential to understanding the quantum-to-classical transition of primordial curvature perturbations during inflation [97], with the superhorizon environment providing the mechanism for classical density perturbations to emerge from quantum fluctuations. These diverse contexts, from laboratory-scale tabletop experiments testing gravity-induced decoherence [98] to the cosmological emergence of classical structure, all require the kind of unified geometric decoherence framework that QPS provides.

The connection between the generalised HPZ model and the QPS formalism suggests that the pointer states identified within the QPS framework naturally emerge from the decoherence dynamics. A rigorous demonstration of this conjecture, solving the generalised master equation in the QPS pointer-state basis and analysing the corresponding decoherence rates, is left for future work. However, the framework established here provides the necessary foundation for such an investigation.

In conclusion, our QPS framework turns the abstract problem of decoherence into a concrete, visual and geometrically sound approach. By integrating the dynamics of pointer state selection and coherence decay into the evolving structure of the QPS, this approach provides a powerful synthetic tool. It not only deepens our fundamental understanding of how quantum systems interact with their environments, addressing foundational questions about the quantum-to-classical transition and the measurement problem, but also may deliver a practical and unified methodology for modelling, mitigating, and potentially harnessing decoherence, particularly in the fields of nanoscience and quantum technologies. Furthermore, the relativistic extension of the QPS framework, with its symmetry group based on linear canonical transformations and its connections to particle physics and cosmology suggests that the geometric approach developed here may have far-reaching implications across physics, spanning the subatomic, nanoscopic and cosmological scales.

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8 References

[1] M. Schlosshauer, “Decoherence, the measurement problem, and interpretations of quantum mechanics,” *Rev. Mod. Phys*., vol. 76, pp. 1267–1305, Feb. 2005, doi: 10.1103/RevModPhys.76.1267.

[2] W. H. Zurek, “Decoherence, einselection, and the quantum origins of the classical,” *Rev. Mod. Phys.*, vol. 75, p. 715, doi: 10.1103/RevModPhys.75.715.

[3] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition*. Springer Science, 2007, doi: 10.1007/978-3-540-35775-9.

[4] R. T. Ranaivoson *et al.*, “Highlighting relations between Wave-particle duality, Uncertainty principle, Phase space and Microstates,” arXiv:2205.08538 [quant-ph], doi: 10.48550/arXiv.2205.08538.

[5] J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement*. Princeton University Press, 1983, doi: 10.1515/9781400854554.

[6] N. T. Manampisoa, R. T. Ranaivoson, R. Raboanary, R. Andriambololona, R. H. M. Ravelonjato and N. Rabesiranana, “Joint momenta-coordinates states as pointer states in quantum decoherence,” 2023, arXiv:2509.18206 [quant-ph], doi: 10.48550/arXiv.2509.18206.

[7] A. A. Tomaz, R. S. Mattos and M. Barbatti, “The quantum measurement problem: a review of recent trends,” *Philos. Mag.*, pp. 1–38, Dec. 2025, doi: 10.1080/14786435.2025.2601922.

[8] E. B. Karlsson, “The Quantum Measurement Problem,” *Quantum Rep*., vol. 7, no. 2, p. 28, Jun. 2025, doi: 10.3390/quantum7020028.

[9] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*. Oxford University Press, 2007, doi: 10.1093/acprof:oso/9780199213900.001.0001.

[10] H. R. Naeij, “Open quantum system approaches to superconducting qubits,” *Quantum Inf. Process.*, vol. 24, p. 220, Jul. 2025, doi: 10.1007/s11128-025-04838-y.

[11] M. Onizhuk and G. Galli, “Colloquium: Decoherence of solid-state spin qubits: A computational perspective,” *Rev. Mod. Phys.*, vol. 97, p. 021001, Apr. 2025, doi: 10.1103/RevModPhys.97.021001.

[12] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe and J. L. O’Brien, “Quantum computers,” *Nature*, vol. 464, pp. 45–53, Mar. 2010, doi: 10.1038/nature08812.

[13] Y.-W. Lu, W. Li and X.-H. Wang, “Quantum and Classical Exceptional Points at the Nanoscale: Properties and Applications,” *ACS Nano*, vol. 19, no. 19, pp. 17953–17978, May 2025, doi: 10.1021/acsnano.4c15648.

[14] A. M. Tyryshkin *et al*., "Electron spin coherence exceeding seconds in high-purity silicon," *Nature Mater.*, vol. 11, pp. 143–147, Dec. 2011, doi: 10.1038/nmat3182.

[15] A. J. Heinrich *et al.*, "Quantum-coherent nanoscience," *Nature Nanotechnol.*, vol. 16, pp. 1318–1329, Nov. 2021, doi: 10.1038/s41565-021-00994-1.

[16] R. Andriambololona, "Mécanique Quantique," Collection Livres Interdisciplines Raoelina Andriambololona (LIRA) Laboratoire de Physique Nucléaire et de Physique Appliquée (LPNPA), Université de Madagascar, 1990.

[17] W. H. Zurek, "Decoherence and the Transition from Quantum to Classical," *Physics Today*, vol. 44, no. 10, pp. 36–44, Oct. 1991, doi: 10.1063/1.881293.

[18] C. A. Brasil and L. A. de Castro, "Understanding the pointer states," *Eur. J. Phys.*, vol. 36, no. 6, p. 065024, Sept. 2015, doi: 10.1088/0143-0807/36/6/065024.

[19] P. Duruisseau, A. Touil and S. Deffner, "Pointer States and Quantum Darwinism with Two-Body Interactions," *Entropy*, vol. 25, no. 12, p. 1573, Nov. 2023, doi: 10.3390/e25121573.

[20] D. A. Chisholm, G. M. Palma and L. Innocenti, "Emergence of quantum Darwinism and pointer states for noncommuting evolutions," *APS Open Sci*., vol. 1, p. 000024, May 2026, doi: 10.1103/75jr-ltct.

[21] W. H. Zurek, "Quantum Theory of the Classical: Einselection, Envariance, Quantum Darwinism and Extantons," *Entropy*, vol. 24, no. 11, p. 1520, Oct. 2022, doi: 10.3390/e24111520.

[22] A. Feller, G. Coeurét Cauquil and B. Roussel, "Einselection from incompatible decoherence channels," *Phys. Rev. A*, vol. 101, p. 062107, Jun. 2020, doi: 10.1103/PhysRevA.101.062107.

[23] W. H. Zurek, S. Habib and J. P. Paz, "Coherent states via decoherence," *Phys. Rev. Lett*., vol. 70, pp. 1187–1190, Mar. 1993, doi: 10.1103/PhysRevLett.70.1187.

[24] W. H. Zurek, "Preferred States, Predictability, Classicality and the Environment-Induced Decoherence," *Prog. Theor. Phys.*, vol. 89, no. 2, pp. 281–312, Feb. 1993, doi: 10.1143/ptp/89.2.281.

[25] D. Dalvit, J. Dziarmaga and W. Zurek, "Predictability sieve, pointer states, and the classicality of quantum trajectories," *Phys. Rev. A*, vol. 72, p. 062101, Dec. 2005, doi: 10.1103/PhysRevA.72.062101.

[26] W. H. Zurek, "Quantum Darwinism," *Nature Phys.*, vol. 5, pp. 181–188, Mar. 2009, doi: 10.1038/nphys1202.

[27] R. Horodecki, J. K. Korbicz and P. Horodecki, "Quantum origins of objectivity," *Phys. Rev. A*, vol. 91, p. 032122, Mar. 2015, doi: 10.1103/PhysRevA.91.032122.

[28] J. K. Korbicz, “Roads to objectivity: Quantum Darwinism, Spectrum Broadcast Structures, and Strong quantum Darwinism – a review,” *Quantum*, vol. 5, p. 571, Nov. 2021, doi: 10.22331/q-2021-11-08-571.

[29] H.-P. Breuer, E.-M. Laine and J. Piilo, “Measure for the Degree of Non-Markovian Behavior of Quantum Processes in Open Systems,” *Phys. Rev. Lett*., vol. 103, p. 210401, Nov. 2009, doi: 10.1103/PhysRevLett.103.210401.

[30] I. de Vega and D. Alonso, “Dynamics of non-Markovian open quantum systems,” *Rev. Mod. Phys*., vol. 89, p. 015001, Jan. 2017, doi: 10.1103/RevModPhys.89.015001.

[31] H.-P. Breuer, E.-M. Laine, J. Piilo and B. Vacchini, “Non-Markovian dynamics in open quantum systems,” *Rev. Mod. Phys*., vol. 88, p. 021002, Apr. 2016, doi: 10.1103/RevModPhys.88.021002.

[32] M. Malekakhlagh, A. Petrescu and H. E. Türeci, “Non-Markovian dynamics of a superconducting qubit in an open multimode resonator,” *Phys. Rev. A*, vol. 94, p. 063848, Dec. 2016, doi: 10.1103/PhysRevA.94.063848.

[33] T. Lacroix, B. W. Lovett and A. W. Chin, “From Non-Markovian Dissipation to Spatiotemporal Control of Quantum Nanodevices,” *Quantum*, vol. 8, p. 1305, Apr. 2024, doi: 10.22331/q-2024-04-03-1305.

[34] K. Brandner, “Dynamics of microscale and nanoscale systems in the weak-memory regime: A mathematical framework beyond the Markov approximation,” *Phys. Rev. E*, vol. 111, p. 014137, Jan. 2025, doi: 10.1103/PhysRevE.111.014137.

[35] M. Odeh *et al.*, “Non-Markovian dynamics of a superconducting qubit in a phononic bandgap,” *Nature Phys.*, vol. 21, pp. 406–411, Jan. 2025, doi: 10.1038/s41567-024-02740-5.

[36] R. T. Ranaivoson, R. Andriambololona, R. Hanitriarivo and R. H. M. Ravelonjato, “Invariant quadratic operators associated with linear canonical transformations and their eigenstates,” *J. Phys. Commun*., vol. 6, no. 9, p. 095010, doi: 10.1088/2399-6528/ac8520.

[37] R. H. M. Ravelonjato, R. T. Ranaivoson, R. Andriambololona, R. Raboanary, H. Rakotoson and N. Rabesiranana, “Quantum and Relativistic Corrections to Maxwell–Boltzmann Ideal Gas Model from a Quantum Phase Space Approach,” *Found. Phys.*, vol. 53, p. 88, Oct. 2023, doi: 10.1007/s10701-023-00727-5.

[38] R. T. Ranaivoson, R. Andriambololona, H. Rakotoson, R. Raboanary, J. Rajaobelison and P. M. Randriantsoa, “Quantum phase space symmetry and sterile neutrinos,” *J. Subat. Part. Cosmol.*, vol. 3, p. 100039, Jun. 2025, doi: 10.1016/j.jspc.2025.100039.

[39] T. G. Philbin, “Generalized coherent states,” *Am. J. Phys*., vol. 82, no. 8, pp. 742–748, Aug. 2014, doi: 10.1119/1.4876963.

[40] B. Bagchi, R. Ghosh and A. Khare, “A pedestrian introduction to coherent and squeezed states,” *Int. J. Mod. Phys*. *A*, vol. 35, no. 19, p. 2030011, 2020, doi: 10.1142/S0217751X20300112.

[41] V. V. Dodonov, “’Nonclassical’ states in quantum optics: a ‘squeezed’ review of the first 75 years,” *J. Opt. B: Quantum Semiclass. Opt*., vol. 4, no. 1, Jan. 2002, doi: 10.1088/1464-4266/4/1/201.

[42] Ó. Rosas-Ortiz, “Coherent and Squeezed States: Introductory Review of Basic Notions, Properties, and Generalizations,” in *Integrability, Supersymmetry and Coherent States. CRM Series in Mathematical Physics*, S. Kuru, J. Negro and L. M. Nieto, Eds., Cham. Springer, 2019, pp. 187–230, doi: 10.1007/978-3-030-20087-9_7.

[43] S. Dey, A. Fring and V. Hussin, “A Squeezed Review on Coherent States and Nonclassicality for Non-Hermitian Systems with Minimal Length,” in *Coherent States and Their Applications: A Contemporary Panorama*, Springer Proceedings in Physics, vol. 205, 2018, pp. 209–242, doi: 10.1007/978-3-319-76732-1_11.

[44] B. L. Hu, J. P. Paz and Y. Zhang, “Quantum Brownian motion in a general environment: Exact master equation with nonlocal dissipation and colored noise,” *Phys. Rev. D*, vol. 45, pp. 2843–2861, Apr. 1992, doi: 10.1103/PhysRevD.45.2843.

[45] J. J. Halliwell and T. Yu, “Alternative derivation of the Hu-Paz-Zhang master equation of quantum Brownian motion,” *Phys. Rev. D*, vol. 53, pp. 2012–2019, Feb. 1996, doi: 10.1103/PhysRevD.53.2012.

[46] G. Homa, J. Z. Bernád and A. Csordás, “Analytical evaluation of the coefficients of the Hu-Paz-Zhang master equation: Ohmic spectral density, zero temperature, and consistency check,” *Phys. Rev. A*, vol. 108, p. 012210, Jul. 2023, doi: 10.1103/PhysRevA.108.012210.

[47] G. Homa, A. Csordás, M. A. Csirik and J. Z. Bernád, “Range of applicability of the Hu-Paz-Zhang master equation,” *Phys. Rev. A*, vol. 102, p. 022206, Aug. 2020, doi: 10.1103/PhysRevA.102.022206.

[48] K. L. Ekinci and M. L. Roukes, “Nanoelectromechanical systems,” *Rev. Sci. Instrum*., vol. 76, no. 6, p. 061101, May 2005, doi: 10.1063/1.1927327.

[49] C. Seoánez, F. Guinea and A. H. Castro Neto, “Dissipation in graphene and nanotube resonators,” *Phys. Rev. B*, vol. 76, p. 125427, Sept. 2007, doi: 10.1103/PhysRevB.76.125427.

[50] J. Schäfer, B. A. Stickler and K. Hornberger, “Decoherence of dielectric particles by thermal emission,” *Phys. Rev. Res.*, vol. 6, p. 043307, Dec. 2024, doi: 10.1103/PhysRevResearch.6.043307.

[51] S. A. Khan, A. A. Afroz and S. S. Ali, “Decoherence Impediments in Quantum Computing and Fundamental Challenges of Quantum Error Correction,” *Asian J. Res. Comput. Sci.*, vol. 18, no. 12, pp. 228–239, Dec. 2025, doi: 10.9734/ajrcos/2025/v18i12800.

[52] T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong and Y. Hirayama, “Coherent manipulation of electronic states in a double quantum dot,” *Phys. Rev. Lett*., vol. 91, p. 226804, Nov. 2003, doi: 10.1103/PhysRevLett.91.226804.

[53] A. Ishizaki and G. R. Fleming, “Unified treatment of quantum coherent and incoherent hopping dynamics in electronic energy transfer: Reduced hierarchy equation approach,” *J. Chem. Phys.*, vol. 130, p. 234111, Jun. 2009, doi: 10.1063/1.3155372.

[54] D. Suter and G. A. Álvarez, “Colloquium: Protecting quantum information against environmental noise,” *Rev. Mod. Phys*., vol. 88, p. 041001, Oct. 2016, doi: 10.1103/RevModPhys.88.041001.

[55] K.-T. Chiang and W.-M. Zhang, “Non-Markovian decoherence dynamics of strong-coupling hybrid quantum systems: A master equation approach,” *Phys. Rev. A*, vol. 103, p. 013714, Jan. 2021, doi: 10.1103/PhysRevA.103.013714.

[56] F. Lastra, S. A. Reyes and S. Wallentowitz, “Analytical model of non-Markovian decoherence in donor-based charge quantum bits,” *J. Phys. B: At. Mol. Opt. Phys*., vol. 44, p. 015504, Dec. 2011, doi: 10.1088/0953-4075/44/1/015504.

[57] A. Ablimit, D. Hitjan and A. Abliz, “Non-Markovian Dynamics of Geometric Quantum Discord in a Double Quantum Dot System,” *J. Low Temp. Phys*., vol. 205, pp. 126–134, Oct. 2021, doi: 10.1007/s10909-021-02621-8.

[58] H.-J. Joo *et al*., “Actively tunable laser action in GeSn nanomechanical oscillators,” *Nat. Nanotechnol*., vol. 19, pp. 1116–1121, Apr. 2024, doi: 10.1038/s41565-024-01662-w.

[59] N. J. Engelsen, A. Beccari and T. J. Kippenberg, “Ultrahigh-quality-factor micro- and nanomechanical resonators using dissipation dilution,” *Nat. Nanotechnol*., vol. 19, pp. 725–737, Mar. 2024, doi: 10.1038/s41565-023-01597-8.

[60] S. Savel’ev, A. L. Rakhmanov, X. Hu, A. Kasumov and F. Nori, “Quantum electromechanics: Quantum tunneling near resonance and qubits from buckling nanoscale bars,” *Phys. Rev. B*, vol. 75, p. 165417, Apr. 2007, doi: 10.1103/PhysRevB.75.165417.

[61] A. Y. Cleland, E. A. Wollack and A. H. Safavi-Naeini, “Studying phonon coherence with a quantum sensor,” *Nat. Commun.*, vol. 15, p. 4979, Jun. 2024, doi: 10.1038/s41467-024-48306-0.

[62] A. B. Bozkurt, O. Golami, Y. Yu, H. Tian and M. Mirhosseini, “A mechanical quantum memory for microwave photons,” *Nat. Phys*., vol. 21, pp. 1469–1474, Aug. 2025, doi: 10.1038/s41567-025-02975-w.

[63] Á. Rivas, S. F. Huelga and M. B. Plenio, “Entanglement and Non-Markovianity of Quantum Evolutions,” *Phys. Rev. Lett*., vol. 105, p. 050403, Jul. 2010, doi: 10.1103/PhysRevLett.105.050403.

[64] H.-P. Breuer, “Foundations and measures of quantum non-Markovianity,” *J. Phys. B: At. Mol. Opt. Phys*., vol. 45, no. 15, p. 154001, Jul. 2012, doi: 10.1088/0953-4075/45/15/154001.

[65] C.-M. Li, N. Lambert, Y.-N. Chen, G.-Y. Chen and F. Nori, “Witnessing Quantum Coherence: from solid-state to biological systems,” *Sci. Rep*., vol. 2, p. 885, Nov. 2012, doi: 10.1038/srep00885.

[66] J. Piilo, S. Maniscalco, K. Härkönen and K.-A. Suominen, “Non-Markovian Quantum Jumps,” *Phys. Rev. Lett*., vol. 100, no. 18, p. 180402, May 2008, doi: 10.1103/PhysRevLett.100.180402.

[67] F. F. Fanchini *et al*., “Non-Markovianity through Accessible Information,” *Phys. Rev. Lett*., vol. 112, no. 21, p. 210402, May 2014, doi: 10.1103/PhysRevLett.112.210402.

[68] S. L. Wu, W. Ma, Z.-M. Wang, P. Brumer and L.-A. Wu, “Steady-state coherences under partial collective non-Markovian decoherence,” *Phys. Rev. Res.*, vol. 7, no. 4, p. 043221, Nov. 2025, doi: 10.1103/nktx-4ltm.

[69] R. T. Ranaivoson, R. Andriambololona, H. Rakotoson and R. Raboanary, “Linear Canonical Transformations in relativistic quantum physics,” *Physica Scripta*, vol. 96, no. 6, p. 065204, Mar. 2021, doi: 10.1088/1402-4896/abeba5.

[70] R. P. Feynman and F. L. Vernon, “The theory of a general quantum system interacting with a linear dissipative system,” *Ann. Phys.*, vol. 24, pp. 118–173, Oct. 1963, doi: 10.1016/0003-4916(63)90068-X.

[71] Á. Rivas and S. F. Huelga, *Open Quantum Systems: An Introduction*. Berlin, Springer, 2012, doi: 10.1007/978-3-642-23354-8.

[72] C. H. Fleming, A. Roura and B. L. Hu, “Exact analytical solutions to the master equation of quantum Brownian motion for a general environment,” *Ann. Phys.*, vol. 326, no. 5, pp. 1207–1258, May 2011, doi: 10.1016/j.aop.2010.12.003.

[73] U. Weiss, *Quantum Dissipative Systems*, 5th ed. World Scientific, 2021, doi: 10.1142/12402.

[74] A. O. Caldeira and A. J. Leggett, “Path integral approach to quantum Brownian motion,” *Physica A*, vol. 121, no. 3, pp. 587–616, Sept. 1983, https://doi.org/10.1016/0378-4371(83)90013-4.

[75] Y.-W. Huang and W.-M. Zhang, “Exact master equation for generalized quantum Brownian motion with momentum-dependent system-environment couplings,” *Phys. Rev. Res.*, vol. 4, no. 3, p. 033151, Aug. 2022, doi: 10.1103/PhysRevResearch.4.033151.

[76] L. Ferialdi and A. Smirne, “Momentum coupling in non-Markovian quantum Brownian motion,” *Phys. Rev. A*, vol. 96, no. 1, p. 012109, Jul. 2017, doi: 10.1103/PhysRevA.96.012109.

[77] S. Bhattacharjee, K. Mandal and S. Sinha, “Decoherence of a charged Brownian particle in a magnetic field: an analysis of the roles of coupling via position and momentum variables,” *J. Phys. A: Math. Theor*., vol. 57, no. 37, p. 375304, Aug. 2024, doi: 10.1088/1751-8121/ad707e.

[78] A. A. Shevyrin, A. K. Bakarov, A. A. Shklyaev and A. G. Pogosov, “Electronic state back action on mechanical motion in a quantum point contact coupled to a nanomechanical resonator,” *Appl. Phys. Lett*., vol. 125, no. 19, p. 192105, Nov. 2024, doi: 10.1063/5.0235485.

[79] N. A. Vaidya and D. H. Santamore, “Effect of noise fluctuation of a quantum tunneling device coupled to a substrate,” *Phys. Rev. B*, vol. 95, no. 15, p. 155439, Apr. 2017, doi: 10.1103/PhysRevB.95.155439.

[80] A. Noguchi, R. Yamazaki, Y. Tabuchi and Y. Nakamura, “Single-photon quantum regime of artificial radiation pressure on a surface acoustic wave resonator,” *Nat. Commun*., vol. 11, no. 1, p. 1183, Mar. 2020, doi: 10.1038/s41467-020-14910-z.

[81] Y. Zhou, X. Xu, Y. Zhang and B. Yao, “Observation of high-order imaginary Poynting momentum optomechanics in structured light,” *Proc. Natl. Acad. Sci*., vol. 119, no. 44, p. e2209721119, Oct. 2022, doi: 10.1073/pnas.2209721119.

[82] G. Burkard, R. H. Koch and D. P. DiVincenzo, “Multilevel quantum description of decoherence in superconducting qubits,” *Phys. Rev. B*, vol. 69, no. 6, p. 064503, Feb. 2004, doi: 10.1103/PhysRevB.69.064503.

[83] G. Marchegiani and G. Catelani, “Nonequilibrium regimes for quasiparticles in superconducting qubits with gap-asymmetric junctions,” *Commun. Phys*., vol. 8, p. 120, Mar. 2025, doi: 10.1038/s42005-025-02052-x.

[84] A. Ranni *et al*., “Decoherence in a crystal-phase defined double quantum dot charge qubit strongly coupled to a high-impedance resonator,” *Phys. Rev. Res*, vol. 6, no. 4, p. 043134, Nov. 2024, doi: 10.1103/PhysRevResearch.6.043134.

[85] F. Ye, A. Ellaboudy, D. Albrecht, R. Vudatha, N. T. Jacobson and J. M. Nichol, “Characterization of individual charge fluctuators in Si/SiGe quantum dots,” *Phys. Rev. B*, vol. 110, no. 23, p. 235305, Dec. 2024, doi: 10.1103/PhysRevB.110.235305.

[86] A. Youssefi, S. Kono, M. Chegnizadeh and T. J. Kippenberg, “A squeezed mechanical oscillator with millisecond quantum decoherence,” *Nat. Phys*., vol. 19, pp. 1697–1702, Aug. 2023, doi: 10.1038/s41567-023-02135-y.

[87] C. Chia, D. Huang, V. Leong, J. F. Kong and K. E. J. Goh, “Hybrid Quantum Systems with Artificial Atoms in Solid State,” *Adv. Quantum Technol*., vol. 7, no. 5, p. 2300461, Apr. 2024, doi: 10.1002/qute.202300461.

[88] A. Isar, “Uncertainty, Entropy and Decoherence of the Damped Harmonic Oscillator in the Lindblad Theory of Open Quantum Systems,” *Prog. Phys*., vol. 47, no. 7, pp. 855–879, Aug. 1999, doi: 10.1002/(SICI)1521-3978(199909)47:7/8<855::AID-PROP855>3.0.CO;2-Z.

[89] P. M. Randriantsoa, R. T. Ranaivoson, R. Andriambololona, R. Raboanary, W. C. Solofoarisina and A. F. H. Rasamimanana, “Geometric structure of the relativistic quantum phase space,” 2026, https://doi.org/10.2139/ssrn.7169885.

[90] A. F. H. Rasamimanana, R. T. Ranaivoson, R. Raboanary, R. Andriambololona, W. C. Solofoarisina and P. M. Randriantsoa, “Contractions of the relativistic quantum LCT group and the emergence of spacetime symmetries,” 2026, arXiv:2603.21333v2 [quant-ph], doi: 10.48550/arXiv.2603.21333.

[91] P. M. Randriantsoa, R. T. Ranaivoson, R. Andriambololona, R. Raboanary, W. C. Solofoarisina and A. F. H. Rasamimanana, “Casimir operators for the relativistic quantum phase space symmetry group,” 2025, arXiv:2512.18262v2 [quant-ph], doi: 10.2139/ssrn.7169885.

[92] S. Donadi and M. Fadel, “Quantum gravitational decoherence of a mechanical oscillator from spacetime fluctuations,” *Phys. Rev. D*, vol. 111, no. 2, p. 026009, Jan. 2025, doi: 10.1103/PhysRevD.111.026009.

[93] I. Pikovski, M. Zych, F. Costa and Č. Brukner, “Universal decoherence due to gravitational time dilation,” *Nat. Phys*., vol. 11, pp. 668–672, Jun. 2015, doi: 10.1038/nphys3366.

[94] D. K. Ferry, L. Huang, R. Yang, Y.-C. Lai and R. Akis, “Open quantum dots in graphene: Scaling relativistic pointer states,” *J. Phys.: Conf. Ser*., vol. 220, p. 012015, Aug. 2010, doi: 10.1088/1742-6596/220/1/012015.

[95] S.-H. Shang and S.-M. Wu, “Quantum steering of GHZ and W states in relativistic motion,” *Eur. Phys. J. C*, vol. 85, no. 7, p. 790, Jul. 2025, doi: 10.1140/epjc/s10052-025-14516-z.

[96] D. L. Danielson, G. Satishchandran and R. M. Wald, “Local description of decoherence of quantum superpositions by black holes and other bodies,” *Phys. Rev. D*, vol. 111, no. 2, p. 025014, Jan. 2025, doi: 10.1103/PhysRevD.111.025014.

[97] F. Sano and J. Tokuda, “False and genuine decoherence in the early universe: a local observer and time-averaged observables,” *J. High Energy Phys*., vol. 2025, no. 7, p. 266, Jul. 2025, doi: 10.1007/JHEP07(2025)266.

[98] S. Bose *et al*., “Massive quantum systems as interfaces of quantum mechanics and gravity,” *Rev. Mod. Phys*., vol. 97, no. 1, p. 015003, Feb. 2025, doi: 10.1103/RevModPhys.97.015003.