

Bayesian Vector Auto-Regression Method as an Alternative Technique for Forecasting South African Tax Revenue

Mojalefa Aubrey Molapo

<https://orcid.org/0000-0002-2179-399X>

University of South Africa

MMolapo@sars.gov.za

John Olutunji Olaomi

<http://orcid.org/0000-0002-1180-3189>

University of South Africa

olaomjo@unisa.ac.za

Njoku Ola Ama

<https://orcid.org/0000-0001-5957-0575>

University of South Africa

University of Botswana

amano@mopipi.ub.bw

Abstract

Tax revenue forecasts are important for tax authorities as they contribute to the budget and strategic planning of any country. For this reason, various tax types need to be forecast for a specific fiscal year, using models that are statistically sound and have a smaller margin of error. This study models and forecasts South Africa's major tax revenues, i.e. Corporate Income Tax (CIT), Personal Income Tax (PIT), Value-Added Tax (VAT) and Total Tax Revenue (TTR) using the Bayesian Vector Auto-regression (BVAR), Auto-regressive Moving Average (ARIMA), and State Space exponential smoothing (Error, Trend, Seasonal [ETS]) models with quarterly data from 1998 to 2012. The forecasts of the three models based on the Root mean square error (RMSE) were from the out-of-sample period 2012Q2 to 2015Q1. The results show the accuracy of the BVAR method for forecasting major tax revenues. The ETS appears to be a good method for TTR forecasting, as it outperformed the BVAR method. The paper recommends that the BVAR method may be added to existing techniques being used to forecast tax revenues in South Africa, as it gives a minimum forecast error.

Keywords: Bayesian Vector Auto-Regression method; Estimation; Forecasting; South Africa; tax revenue types



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Introduction

Tax revenue is the government's key source of income, and the government needs to estimate its expenditure before it can budget for an income to meet its obligations. One of the key issues in the design of sound fiscal policy has been the accuracy of budget forecasts, particularly tax revenue forecasts (Nandi, Chaudhury, and Hasan 2015). Accurate revenue forecasting is crucial in meeting expenditure for budgeting, particularly those of tax revenues (Auerbach 1999).

The contribution of tax revenue to the fiscus highlights the need for tax authorities to consider using a combination of methods to improve forecast accuracy. The common methodologies used by tax authorities are based on growth trends, averages and contributions of the previous period, as well as the expert knowledge. Shahnazarian, Solberger and Spanberg (2017), citing Jenkins, Kuo and Shukla (2000), list five main methods:

- i) The extrapolation of tax revenue method, which uses ARIMA models to estimate the development of tax revenue.
- ii) The underlying tax development method, which estimates the “structural” or “underlying” tax base, after which information on tax rules, legislation and tax behaviour are used to calculate the underlying tax revenue.
- iii) The auditing method, which uses the difference between the calculated tax and extra tax paid by the firm when the tax is settled on the audit day to make assessments about the tax revenue level.
- iv) The elasticity method, which is a conditional projection, where the future tax revenue is calculated based on a starting point, combined with an estimate of the ratio of the change in tax revenues and the change in the macroeconomic variable (see Wolswijk 2007).
- v) The macroeconomic regression models, which estimate functional relationships between sets of macroeconomic variables and the tax revenue in question.

Baghestani and McNown (1992) have shown that models—like integrated auto-regressive models such as Auto-regressive Integrated Moving Average (ARIMA) models, co-integrated VAR models, and error-correction models—in general, have good predictive abilities compared with official forecasts. Krol (2010) compared the performance of BVAR and VAR, and found that BVAR performs better based on the Root mean square error (RMSE) forecasting criterion. Similarly, Shahnazarian et al. (2017) indicate that BVAR models are robust and produce reasonable conditional forecasts when compared with Direct Tax Revenue forecasts from a Mixed Data Sampling (MIDAS) equation and typical naïve forecasts from Simple Integrated AR models with exogenous variables (ARIX).

There is a need for tax authorities to use various methodologies to set revenue targets accurately. One of the ways of determining the best approach is to compare the precision of different techniques. According to the South African Revenue Service (SARS) Annual Report for 2014/15, revenue estimates for the medium term were set or adjusted on three occasions during the financial year. Estimates were announced in the February 2014 Budget (generally referred to as the Printed Estimate), in October 2014 in the Medium Term Budget Policy Statement (MTBPS), and in the February 2015 Budget (the Revised Estimate). It was observed that some of the errors of the printed estimates were more than five per cent.

Based on the robustness of the BVAR method in forecasting literature, which is not currently in use at SARS, this study compares the performance of BVAR with ARIMA and ETS (both currently among the methods employed in South Africa), in forecasting the main tax types in South Africa. The best technique is recommended to complement the existing methodologies used at SARS, thus assisting in reducing or eliminating the frequency of revising tax revenue estimates due to large errors.

Tax analysis is complex and impacted by many factors, which makes it impractical for tax forecasting to depend on only one approach. Such factors may include the tax rates, taxpayer compliance and behaviour, tax structures, tax efficiency, tax morality, and the efficacy of social contract, amongst many others. In addition, tax revenue forecasts are developed in relation to economic theories, employed forecasting techniques and most importantly, assumptions derived from economic variables such as growth in the national income, rate of inflation, interest rates, consumption, employment and the international economic/political environment (Jenkins et al. 2000). The performance of tax revenue is ultimately dependent on the performance of the economy (SARS 2004).

Various studies frequently use ARIMA models. The study conducted by Nazmi and Leuthold (1985) developed a time-series model for predicting state income tax receipts using the Hannan-Quinn criteria. The authors determined the linear and log-linear versions of the ARIMA (1,0,0) model and used a Box-Cox transformation to select a linear version of a time-series model. In their study, Meylar, Kenny and Quinn (1998) outlined the steps required in order to use ARIMA time-series models for projecting Irish inflation. The Box Jenkins techniques and the Objective Penalty function methods were suggested as ways to identify an ARIMA model. Legeida and Sologoub (2003) developed a suitable ARIMA model for predicting VAT revenue in the short-run, and the forecast was consistent with government projections for the budget. Koirala (2011) also used the ARIMA technique as one of the tools to forecast government revenues. The study conducted by Mehmood and Ahmad (2012) aimed to forecast Pakistan's exports to SAARC for the years ahead using an ARIMA model. Dadzie (2013) used ARIMA models to forecast the domestic and import VAT of Ghana using data from 1999 to 2009. Zakai (2014) modelled Pakistan's GDP using a set of ARIMA models based on the Box-Jenkins technique. Guizzi, Silvestri, Romano, and Revetria (2015) analysed and forecast temperature, pressure and humidity using four years of time-series

data. Skarbøvik (2013) employed the AR process, ARIMA process and an exponential smoothing state space (ETS) model to find an appropriate fit for projecting residential house prices in Norway. Huselius and Walled (2014) compared the performances of univariate time-series methods in predicting the Swedish inflation rate. The authors fitted Exponential smoothing and ARIMA models, both regular and underlying state space models and the forecasts compared with those of the National Institute of Economic Research (NIER). In modelling and forecasting fish catches, Bako (2014) developed the state space approach (ETS). The author used the Box-Jenkins method and the ETS state space exponential method to predict the fish catch of three commercial fish species found in Malaysian waters.

Ramos (1996) showed that BVAR provides important information for the people who are responsible for marketing, by utilising impulse response functions and decompositions of variance. In another study, Yao (2011) employed Bayesian VAR methods, as proposed by Litterman (1986), to estimate and forecast several North Dakota macroeconomic variables, including employment, income and tax receipts. Spulbăr, Nitoi and Stanciu (2012) also used the BVAR model in Romania to provide an analysis of the transmission mechanism of the monetary policy in the country, while Shahnazarian et al. (2017) employed BVAR to forecast and analyse corporate tax revenues in Sweden.

In South Africa, the national treasury under leadership of the Minister of Finance, deals with the issues of tax and tax legislation. The South African Revenue Service (SARS) is the revenue authority given the mandate to collect and manage all taxes, duties and levies. Other functions of SARS, in terms of the SARS Act No. 34 of 1997, include ensuring maximum compliance with tax and customs legislation, and providing customs services that will maximise revenue collection, protect our borders and facilitate trade (SARS Annual Report, 2015).

Prior to 2001, South African taxpayers were taxed based on source taxation. The tax system in South Africa is residence-based, meaning residents are qualified to get certain exclusions. The residents are taxed on their income and capital gains acquired domestically and globally, regardless of where their income was earned. The taxpayers who are not residents of South Africa are taxed on their income gained from a source in South Africa. Taxes from outside the borders of South Africa are credited against tax payable on foreign income. The introduction of income tax in South Africa can be traced back to 1914 with the Income Tax Act No. 28. Through the years, the Income Tax Act has undergone numerous changes, and the Act currently adopted is the Income Tax Act No. 58 of 1962. The Act outlines provisions for four different types of income tax, namely normal tax, donations tax, secondary tax on companies, and withholding tax. The income tax system in South Africa is progressive and is based on the principle that wealthy people should contribute a greater share of tax to the state than the poor.

Three major tax types, the Personal Income Tax (PIT), the Corporate Income Tax (CIT) and the Value-Added Tax (VAT) dominate the tax structure of South Africa, contributing approximately 80 per cent, and the remaining tax types collectively contribute 20 per cent to the total tax collection (see www.sars.gov.za for all tax types in South Africa). PIT is the largest source of tax revenue in South Africa. For the past five years, PIT collections have been contributing 34.4 per cent on average to total tax collection. VAT collections contribute about 25.8 per cent while CIT collections contribute about 19.8 per cent on average to total tax revenue (Treasury SA 2015).

National governments, in the course of budget preparation (Golosov and King 2002), make revenue forecasts. “Because of the magnitude of the fiscal problems facing many states, forecasting has assumed a more central role in the policy-making process; as a result, revenue forecasts are closely examined and accuracy is essential for planning purposes” (Fullerton 1989). In developing a fiscally sound budget, there is a need to generate reliable forecasts with good precision and to use forecasts as a benchmark for how much money the government needs to be able to provide services to its citizens.

Accurate revenue forecasts are widely regarded as a key element for the design and execution of sound fiscal policies (Danninger 2005). Most governments use revenue forecasts to set targets, and in turn, use targets as a performance measure. “At the micro level, realistic revenue forecasts become effective standards of measurement against which the actual performance of collecting agencies is assessed” (Gamboa 2002). Setting a target combines various processes, ranging from the development of revenue forecasting models to experts’ knowledge on various tax types, and consultations with other external stakeholders.

Tax authorities are faced with the challenges of producing accurate tax revenue forecasts, which is an important part of the government’s budget process. To achieve accurate revenue collection forecasts, governments should employ various sound statistical techniques and compare model performances based on their forecasting powers and a smaller margin of error. The accuracy of the statistical model will encourage revenue authorities to explore various statistical methodologies as an alternative approach to existing methods.

This paper models and forecasts South Africa’s major tax revenues of CIT, PIT, VAT and Total Tax Revenue (TTR), using Bayesian Vector Auto-regression (BVAR), Auto-regressive Moving Average (ARIMA) and State Space exponential smoothing (Error, Trend, Seasonal [ETS]) models, with quarterly data from 1998 to 2012Q1. The forecasts of the three models, based on Root mean square error (RMSE) forecasting accuracy measure, were from the out-of-sample period 2012Q2 to 2015Q1. Comparisons of the performance (forecasting accuracy) of the BVAR approach with the ARIMA and ETS methods are made in order to recommend the best model for forecasting tax revenue in South Africa.

Methodology

Forecasting methodologies can be classified according to two broad approaches, namely time-series forecasting and econometric forecasting. Time-series forecasting predicts the variable values from previous observations of that variable, while econometric forecasting is based on models that relate the endogenous variable to a number of exogenous variables with residuals considerations.

The tools, which most developed countries use to forecast various tax revenues, consist of macro-based models (Chun-Yan Kuo 2000). These models specify the proxies for tax types in order to determine the potential revenue collection for each tax type. The methods are based on the past performance of tax collections and economic growth. In generating the revenue forecasts, discretionary changes (revenue initiatives and legislative changes) should be taken into account by adjusting them to consider only the revenue collection associated with economic performance. The discretionary effects are not adjusted in this study due to a lack of distinction between revenue collection related purely to economic performance and collection linked to fiscal policies.

Both time-series and econometric approaches are used in this study to compare performance and the accuracy of forecasts using the out-of-sample data. The three methods used are the ARIMA, the ETS and the BVAR.

The ARIMA method, also known as the Box-Jenkins method, is a commonly used technique in forecasting. The general form of an ARIMA(p,d,q)x(P,D,Q)_s model for a time series Z_t with white noise distributed innovations (also called errors, shocks or residuals) ϵ_t is:

$$\Phi_P(B^S)\varphi_p(B)(1-B)_S^D(1-B)^dZ_t = \Theta_Q(B^S)\theta_q(B)\epsilon_t$$

where $\Phi_P(B^S)$ and $\Theta_Q(B^S)$ are the seasonal characteristic functions of the auto-regressive and moving average polynomials of order P and Q respectively, with s time span of repeating seasonal pattern. $\varphi_p(B)$ and $\theta_q(B)$ are the non-seasonal characteristic functions of the auto-regressive and moving average polynomials of order p and q respectively. $(1-B)$ is the difference operator, while d and D are the order of non-seasonal and seasonal differencing employed to achieve stationarity respectively, with $B^kZ_t = Z_{t-k}$. The ARIMA method provides a means of model identification, parameter estimation, and forecasting. ARIMA modelling takes into account trends, seasonality, cycles, shocks and non-stationary aspects of a data set when making forecasts. Many software programs, like E-Views, have a standard package for a time-series analysis using the ARIMA method. Hyndman and Athanasopoulos (2013) expounds on a general method for forecasting, using the ARIMA model. We employ the automatic ARIMA forecasting in E-Views software to generate 20 models with the best top five models selected for each tax type analysed.

Exponential smoothing ETS (Error, Trend, Seasonal) is an improved version of the exponential smoothing methods. Hyndman, Koehle, Snyder and Grose (2002) developed state space models for smoothing methods of which prediction intervals, maximum likelihood estimation and Akaike's Information Criterion may be calculated. The state space models generate automatic forecasts with less human interaction. The state space model is made up of the equation that describes the observed data and some transition equations that describe how the unobserved components or states, like level (ℓ_t), trend (b_t), and seasonality (s_t) change over time. There are 30 ETS models, 15 corresponding to models with multiplicative errors and the other 15 with additive errors (Hyndman and Athanasopoulos 2013). The idea of exponential smoothing methods is to produce forecasts using weighted averages of past observations, with the weights decaying exponentially as the observations get older. This suggests that the more recent the observation, the higher the associated weight.

There are several categories of exponential smoothing methods, the simplest of which is Simple Exponential Smoothing (SES). SES is appropriate for generating forecasts of data with no trend or seasonal pattern. Holt's linear trend methods involve a forecasting equation and two smoothing equations for the level and the trend, as given by the equations below:

$$\begin{aligned}\tilde{y}_t &= \alpha y_t + (1 - \alpha)(\tilde{y}_{t-1} + r_{t-1}); & \text{where, } 1 > \alpha > 0 \\ r_t &= \beta(\tilde{y}_t - \tilde{y}_{t-1}) + (1 - \beta)r_{t-1}; & \text{where, } 1 > \beta > 0 \\ \hat{y}_{t+1} &= \tilde{y}_t + lr_t\end{aligned}$$

\tilde{y}_t is the level estimate of the series at time t and r_t represents the slope estimate of the series at time t. Series smoothness is determined by two parameters, α and β ; these parameters must lie between 0 and 1.

The Holt-Winters seasonal method has a forecast equation and three smoothing equations, one for the level (ℓ_t), one for the trend (b_t), and one for the seasonality component (S_t), with associated smoothing parameters α , β and γ respectively. The three smoothing equations for multiplicative seasonality are given as follows (Makridakis and Wheelwright 1998):

$$\begin{aligned}L_t &= \alpha \frac{y_t}{s_{t-s}} + (1 - \alpha)(L_{t-1} + b_{t-1}) & \text{where, } 1 > \alpha > 0 \\ b_t &= \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1} & \text{where, } 1 > \beta > 0 \\ s_t &= \gamma \frac{y_t}{L_t} + (1 - \gamma)s_{t-s} & \text{where, } 1 > \gamma > 0 \\ \hat{y}_{t+m} &= (L_t + b_t m)s_{t-s+m}\end{aligned}$$

where s represents the length of seasonality, s_t denotes the seasonal component and \hat{y}_{t+m} is the forecast for m periods in the future.

The seasonal methods may be additive or multiplicative, depending on seasonal variations, whether constant or changing in proportion to the level of the series. Hyndman, Koehler, Ord and Snyder (2008) showed that all exponential smoothing methods are optimal forecasts from innovations state space models. The combination of the trend and seasonal components results of 15 possible exponential smoothing methods are as shown in Table 1.

Table 1: Classification of exponential smoothing methods

Trend component	Seasonal component		
	N	A	M
	(None)	(Additive)	(Multiplicative)
N (None)	N,N	N,A	N,M
A (Additive)	A,N	A,A	A,M
A_d (Additive damped)	A _d ,N	A _d ,A	A _d ,M
M (Multiplicative)	M,N	M,A	M,M
M_d (Multiplicative damped)	M _d ,N	M _d ,A	M _d ,M

Source: Hyndman and Athanasopoulos (2013). *Forecasting Principles and Practice*

Yusof and Kane (2012) submitted that “The versatile and fully automatic ETS framework requires neither stationarity nor strict linearity to produce contemporaneous time-series for variable time horizons.” A complete and detailed explanation of ETS models is found in Hyndman, Koehler, Ord and Snyder (2005).

BVAR is widely used to forecast economic variables, but there is very little research on its usage in forecasting tax revenue. The BVAR model is Vector Auto-regression (VAR) with priors introduced to control coefficients of the variables. The VAR model is a multi-equation system where all the variables are treated as endogenous (dependent).

The VAR (p) model is:

$$\mathbf{Y}_t = \mathbf{a} + \mathbf{A}_1 \mathbf{Y}_{t-1} + \mathbf{A}_2 \mathbf{Y}_{t-2} + \cdots + \mathbf{A}_p \mathbf{Y}_{t-p} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{Y}_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$ is an $(n \times 1)$ vector of time-series variables, \mathbf{a} is an $(n \times 1)$ vector of intercepts, $\mathbf{A}_i, (i = 1, 2, \dots, p)$ is an $(n \times n)$ coefficient matrices and $\boldsymbol{\varepsilon}_t$ is an $(n \times 1)$ vector of unobservable *i.i.d* $\sim N(\mathbf{0}, \boldsymbol{\Sigma})$.

Different choices of priors may be used with the VAR models; in this paper, three priors were used—the first was the Minnesota prior (Litterman 1986). This prior assumes that $\boldsymbol{\Sigma}$ is known and equal to $\hat{\boldsymbol{\Sigma}}$. The $\boldsymbol{\beta}$ prior is distributed as $\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_0, V)$ with mean $\boldsymbol{\beta}_0 = \mathbf{0}$ and Covariance $V \neq 0$. The Minnesota prior is set as:

$$V_{ij} = \begin{cases} a_1/h^2 & \text{for coefficients on own lag } h, \text{ for } h = 1, \dots, p \\ a_2 \sigma_{ii} / h^2 \sigma_{jj} & \text{for coefficients on own lags of variables } i \neq j \\ a_3 \sigma_{ii} & \text{for coefficients on exogenous variables} \end{cases}$$

where σ_i^2 is the i^{th} diagonal element of $\boldsymbol{\Sigma}$.

The second prior, suggested by Sims and Zha (1998) is:

$$\Pi(A) = \Pi(A_0) \Pi(A_+ | A_0) = \Pi(A_0) \Phi(B_0, \Psi_0)$$

where, Π denotes a marginal distribution of A_0 , and Φ represents the standard normal density with mean B_0 and covariance Ψ_0 . The elements of Ψ_0 is written as:

$$\Psi_{0l,ij} = \left(\frac{\lambda_0 \lambda_1}{\sigma_j l^{\lambda_3}} \right)^2 \quad i, j = 1, \dots, n.$$

where σ_j^2 is the j^{th} element of $\boldsymbol{\Sigma}$ for the l^{th} lag of variable i in equation j . The general beliefs about the series are given by hyper-parameters λ_0, λ_1 and λ_3 .

The third prior used was Normal Wishart, which assumes that the fixed and diagonal variance-covariance matrix of residuals is relaxed. The conjugate prior for normal data is:

$$p(\phi | \Sigma) = N(\bar{\phi}, \Sigma \otimes \bar{\sigma})$$

$$p(\Sigma) = iW(\Sigma, \alpha)$$

The prior distribution of ϕ will be normal with prior mean $E(\phi) = \bar{\phi}$ and prior variance $V(\phi) = (\alpha - n - 1)^{-1} \bar{\Sigma} \otimes \bar{\sigma}$, where α is the degree of freedom of the inverse-Wishart satisfying $\alpha > n + 1$.

From Bayes rule, the posterior is:

$$p(\phi | \Sigma, Y) = N(\tilde{\phi}, \Sigma \otimes \bar{\sigma})$$

where

$$\tilde{\phi} = (\bar{\phi}^{-1} + X'X)^{-1}$$

$$\tilde{\Sigma} = \hat{B}'_{ols} X' X \hat{B}_{ols} + \bar{B}' \bar{\phi}^{-1} \bar{B} + \bar{\Sigma} + (Y - X \hat{B}_{ols})' (Y - X \hat{B}_{ols}) - \tilde{B}' (\bar{\phi}^{-1} + X'X) \tilde{B}$$

and

$$\tilde{B} = \tilde{\phi} (\bar{\phi}^{-1} \bar{B} + X' X \hat{B}_{ols})$$

Litterman's (1986) assumption was normal prior distribution with a mean of zero and small standard deviation, while the mean on a variable's first own lag is one with a larger standard deviation.

Forecasting Performance Measures

In evaluating the accuracy of forecasts, frequently used measures include Mean Square Error (MSE), Root Mean Square Error (RMSE) and Mean Absolute Per cent Error (MAPE). Akaike Information Criterion (AIC), Schwartz Bayesian Information Criterion (BIC), Hannan Quinn Information Criterion (HQC), and Log likelihood (LL) amongst many others, are usually used for the model evaluation. In this study, RMSE, AIC, BIC, and HQC are used for models' evaluation and comparison. The measures are defined as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2}$$

where \hat{y}_t is the forecast value in the period t , y_t is the actual value in the period t , and n is the size of the sample.

$$AIC = -2 LL + 2 m$$

$$BIC = -2 LL + m \log n$$

$$HQC = -2 LL + 2m \log (\log n),$$

where m is the degree of freedom, n is the sample size and LL is the Log-likelihood function of the parameter of interest.

The model is best when any of the measures has the least (minimum) value.

Data Collection

The data used in the study consists of monthly total tax collections of PIT, CIT, VAT and TTR in South Africa. For the purpose of this study, PIT includes assessed tax, provisional tax and PAYE collected by employers on behalf of employees less refunds. PIT has a direct relationship with employment and usually benefits from above-inflation wage settlements, bonuses paid out, retrenchment packages, and once-off PAYE collections from the vesting of shares. Therefore, the PIT data series is unadjusted for these factors, which may result in large model errors.

CIT revenue comprises of assessed and provisional payments paid by corporate organisations minus the refunds. CIT is levied on profit made by companies and as it is affected by economic performance, poor economic conditions have an impact on it.

The VAT collection data series used in this study is only domestic, excluding import VAT. Various factors affect VAT, such as consumer spending caused by high consumer debt, modest employment, and low growth in disposable income.

The tax revenue data is sourced from the annual SARS tax statistics publication and annual reports. The monthly tax collection data is converted into quarterly data in order to match its counterpart economic data. The quarterly economic data is used specifically for the BVAR technique, which requires dependent variable (tax revenue) and independent or explanatory variables (economic data), such as: Gross Domestic Product (GDP) and Consumer Price Index (CPI) as explanatory variables for TTR, Gross Operating Surplus (GOS) and Rand/dollar (Randol) exchange rate as explanatory variables for Corporate Income Tax (CIT). The chosen explanatory variables for Value-Added tax are Gross Domestic Expenditure (GDE) and Private Consumption Expenditure (PCE), while those for Personal Income Tax (PIT) are Compensation of

Employee (CoE) and Employment (Empl). We chose these variables based on economic theory and the literature. Economic data was sourced from Statistics South Africa (STASSA) and the South African Reserve Bank's (SARB) online statistics tools.

Results

The natural logarithm of the variables was modelled to smoothen and lessen the severity of the data. Each of the variables is plotted to show its stationarity or otherwise. The time plot of the quarterly CIT data is as shown in Figure 1. It is evident that the CIT time series displays a non-stationary pattern. Other variables display similar patterns of non-stationarity.

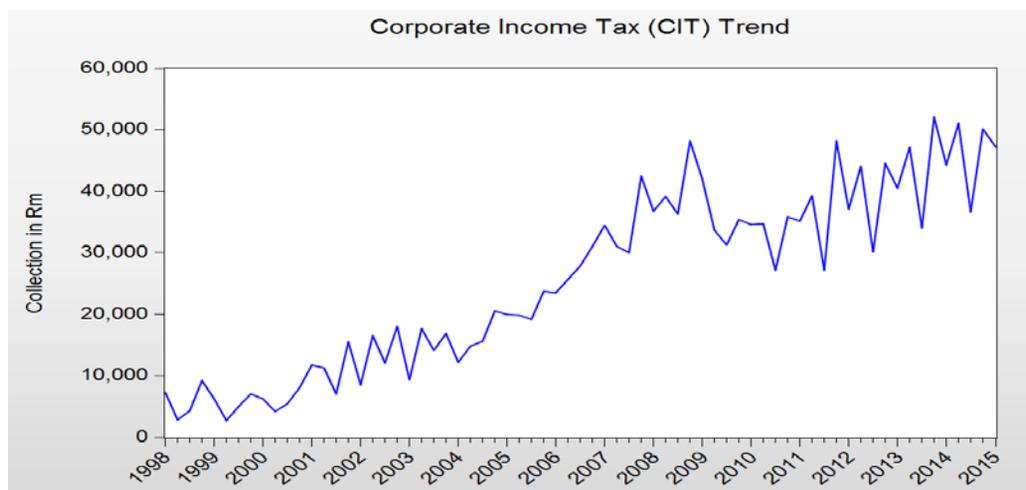


Figure 1: Corporate income tax trend from 1998–2014

Assessment of the logarithm of variables for stationarity is made using the Augmented Dickey-Fuller (ADF) and Phillip Peron (PP) tests. Table 2(a & b) show the results of the tests. It is evident from Table 2(a & b) that the variables became stationary at their first difference.

Table 2a: ADF and PP tests results for stationarity of individual time series at level

Variables	Augmented Dicky-Fuller (ADF)		Phillips-Peron (PP)	
	Constant	Constant & Trend	Constant	Constant & Trend
LCIT	-1.778	-0.565	-1.680	-4.986***
LPIT	1.682	-2.197	0.352	-7.691***
LVATP	-0.610	-1.768	-0.271	-6.977***
LTTR	-1.031	-2.224	-0.021	-5.807***
LGOS	-2.880*	-1.260	-3.418***	-1.769
LRandol	-1.829	-2.163	-1.898	-2.211
LCoE	-0.480	-1.759	-0.745	-5.531***
LEmpl	-0.878	-1.566	-0.878	-1.561
LGDE	-1.018	-0.750	-1.891	-3.957
LPCE	-1.841	-0.615	-1.672	-6.286***
LGDP	-2.581	-0.087	-3.375**	0.873
LCPI	-1.500	-6.073***	-1.164	-6.263***

***; **, * denotes stationarity at 1%, 5% and 10% respectively

Table 2b: ADF and PP tests results for stationarity of individual time series at first difference

Variables	Augmented Dicky-Fuller (ADF)		Phillips-Peron (PP)	
	Constant	Constant & Trend	Constant	Constant & Trend
DLCIT	-13.795***	-14.068***	-23.127*	-35.102***
DLPIT	-3.712*	-4.294*	-29.596*	-34.100***
DLVATP	-5.298***	-5.279***	-31.128*	-33.350***
DLTTR	-2.880**	-2.956	-18.072*	-18.308***
DLGOS	-1.339	-2.954	-9.090*	-13.108***
DLRandol	-6.535***	-6.483***	-6.549*	-6.496***
DLCoE	-2.897**	-2.865	-18.665*	-18.450***
DLEmpl	-8.760***	-8.707***	-8.759*	-8.707***
DLGDE	-3.838***	-3.959**	-17.495*	-18.332***
DLPCE	-1.680	-2.415	-24.018*	-30.784***
DLGDP	-1.495	-2.896	-8.657*	-10.345***
DLCPI	-6.473***	-6.566***	-23.146*	-29.491***

***; **, * denotes stationarity at 1%, 5% and 10% respectively

Modelling and Model Selection

Each of the LCIT, LPIT, LVAT and LTTR is modelled based on the three chosen techniques of Minnesota prior, Normal Wishart prior and Sims-Zha prior for BVAR. Five different ARIMA and ETS models were developed and the best model selected based on the AIC criterion. The RMSE criterion is used in selecting the best BVAR model.

In this paper, only the CIT variable analysis is fully discussed. The results for all other variables are just summarised. Complete analysis for all the variables is available in Molapo (2018). The computations follow the same process as that of CIT.

CIT ARIMA Model Identification and Estimation

CIT ARIMA Estimation

Having stationarised the series, correlogram plots of the stationary variables were charted. Figure 2 shows the correlogram of DLCIT. The correlogram is used to determine the parameters (p, q) of an ARIMA (p,d,q) model.

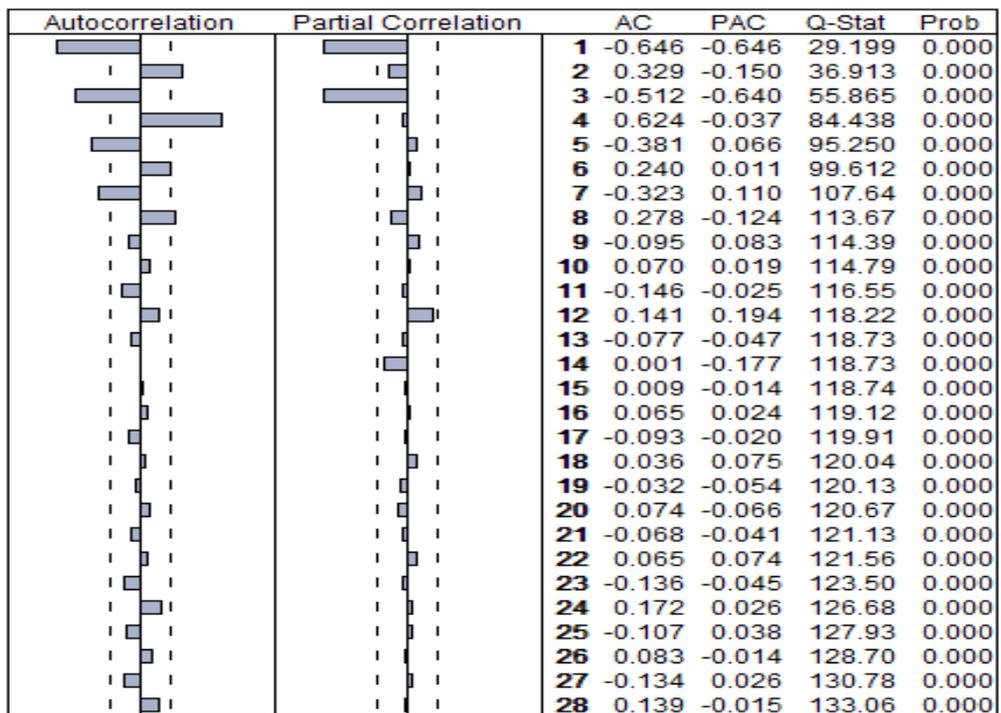


Figure 2: Correlogram of CIT (Logarithmic Form, 1st Differenced) (DLCIT)

Figure 2 shows that the ACF cuts off at lag 4 ($q = 4$) and the PACF cuts off at lag 3 ($p = 3$). Now the range of models $\{ARMA(p, q) : 0 \leq p \leq 3, 0 \leq q \leq 4\}$ are explored, and the best model is selected based on AIC and BIC. After identifying the parameters, the automatic ARIMA forecasting was performed using the Eviews software. Twenty models were generated and the top five ARIMA models are shown in Table 3.

Table 3: Top five CIT ARIMA models based on AIC.

Model	LogL	AIC*	BIC	HQ
(4,1,1)	1.6012	0.0500	0.1585	0.0920
(3,1,0)(1,0,1)₄	2.7467	0.1142	0.3292	0.1977
(4,1,0)(1,0,0)₄	2.4979	0.1229	0.3379	0.2065
(3,1,0)(0,0,1)₄	1.2703	0.1309	0.3101	0.2005
(2,1,0)(1,0,1)₄	1.1029	0.1367	0.3160	0.2064

The appropriate model selected is thus ARIMA (4,1,1). The identified model is given as:

$$W_t = \beta_1 W_{t-1} + \beta_2 W_{t-2} + \beta_3 W_{t-3} + \beta_4 W_{t-4} - \theta_1 \varepsilon_{t-1} + \varepsilon_t;$$

$$W_t = Y_t - Y_{t-1}$$

and the significant model is $\nabla DLCIT_t = 0.7228 \nabla DLCIT_{t-4} + 0.6901 \varepsilon_{t-1} + \varepsilon_t$.

The chosen ARIMA model is stable as the inverse roots of the characteristic polynomials are not outside the unit circle and the correlogram of the residual indicated a white noise. These are depicted in Figures 3 and 4. In addition, the Jarque-Bera normality test confirms that the residual of ARIMA (4,1,1) model follows a normal distribution, since its p-value is 0.1275.

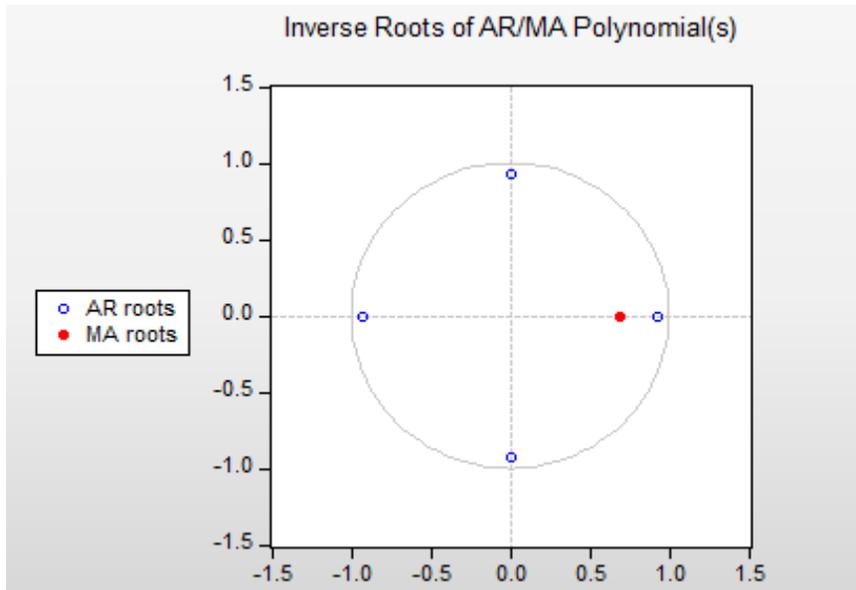


Figure 3: Inverse Roots of AR and MA Process for ARIMA (4,1,1)

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 -0.045	-0.045	0.1207	
		2 0.004	0.002	0.1216	
		3 0.012	0.012	0.1301	0.718
		4 0.016	0.017	0.1454	0.930
		5 -0.022	-0.020	0.1749	0.982
		6 0.128	0.126	1.2367	0.872
		7 -0.118	-0.109	2.1632	0.826
		8 -0.255	-0.272	6.5651	0.363
		9 0.170	0.161	8.5537	0.286
		10 -0.067	-0.055	8.8671	0.354
		11 -0.202	-0.233	11.818	0.224
		12 -0.167	-0.214	13.884	0.178
		13 -0.141	-0.155	15.395	0.165
		14 -0.061	-0.025	15.685	0.206
		15 0.270	0.207	21.449	0.065
		16 0.025	0.039	21.502	0.089
		17 -0.104	-0.013	22.402	0.098
		18 -0.009	-0.096	22.409	0.130
		19 0.034	-0.093	22.511	0.166
		20 -0.030	-0.098	22.591	0.207
		21 0.107	0.010	23.656	0.210
		22 0.068	0.111	24.099	0.238
		23 -0.016	0.058	24.123	0.287
		24 0.155	-0.003	26.577	0.228

Figure 4: Correlogram of the Residuals of CIT ARIMA (4,1,1) Model

CIT ARIMA Model Forecasts

The forecasts of CIT values for the next 12 quarters, from the second quarter of 2012 to the first quarter of 2015, was made using the selected ARIMA (4,1,1) model.

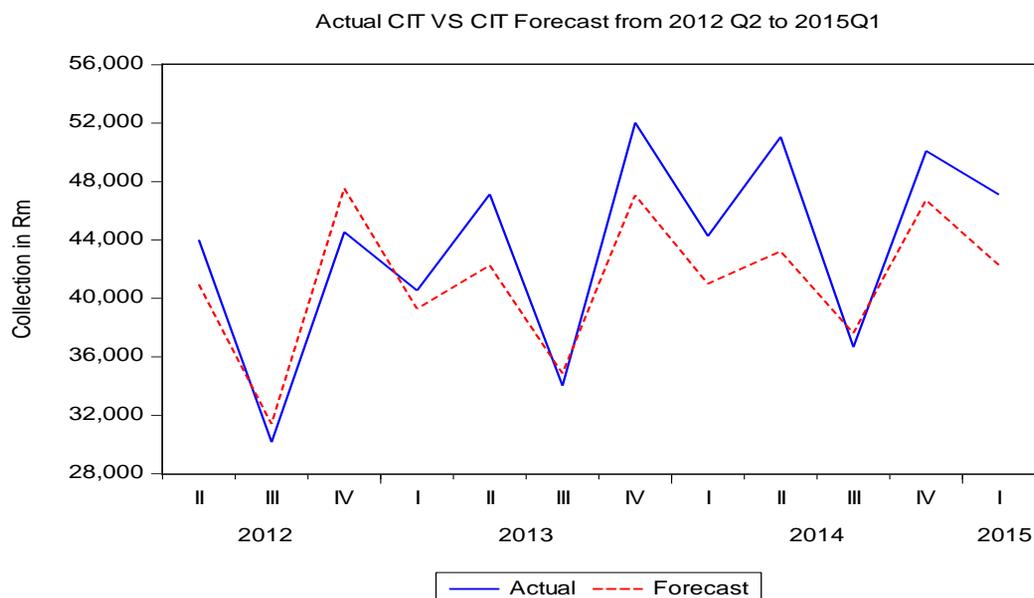


Figure 5: Forecasts for h=12 quarters ahead with an ARIMA (4,1,1) model

Figure 5 shows the diagrammatic representation of the quarterly actual CIT collection in rand million and its forecast. Checking the measures of forecast accuracy, the RMSE is 3847.81; MAE of 3286.44, MAPE is 7.12 while Theil statistics is 0.05.

CIT Error, Trend, Seasonal Models

CIT ETS Model Selection

The ETS models were executed by using E-views package Automatic Forecast tools, which produced 30 models, and the best model was selected based on AIC. Out of the 30 model specifications, the Multiplicative error, Additive trend, Additive seasonal (M, A, A) model is the best and represented by the equation:

$$\begin{array}{l}
y_t = (\ell_{t-1} + b_{t-1} + s_{t-m})(1 + \varepsilon_t) \\
\ell_t = \ell_{t-1} + b_{t-1} + \alpha(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t \\
b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t \\
s_t = s_{t-m} + \gamma(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t
\end{array}
\quad \text{or} \quad
\begin{array}{l}
y_t = (\ell_{t-1} + b_{t-1} + s_{t-m})(1 + \varepsilon_t) \\
\ell_t = \ell_{t-1} + b_{t-1} + 0.35(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t \\
b_t = b_{t-1} + 0(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t \\
s_t = s_{t-m} + 0.8(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t
\end{array}$$

The selected model based on the AIC criterion has a level smoothing parameter estimate $\alpha = 0.35$, trend parameter $\beta = 0$ (zero indicates that the trend components do not change from its starting value) and the seasonal parameter $\gamma = 0.80$. Table 4 shows the best five ETS models based on the AIC criterion.

Table 4: The Top five CIT ETS models based on AIC

<i>Model</i>	<i>Likelihood</i>	<i>AIC*</i>	<i>BIC</i>	<i>HQ</i>
(M, A, A)	-545.1140	1174.9200	1191.2700	1181.2700
(M, M _D , A)	-544.2740	1175.2400	1193.6300	1182.3900
(M, A _D , A)	-545.1140	1176.9200	1195.3100	1184.0700
(M, M, A)	-546.2030	1177.1000	1193.4500	1183.4500
(M, A, M)	-551.7960	1188.2900	1204.6300	1194.6400

CIT ETS Model Forecasts

The best ETS (M, A, A) model is used to generate the CIT forecasts as plotted in Figure 6. The graph shows that the forecasts for the CIT series are closer to the actual series, except in quarter four of 2012 and quarter four of 2015. The closeness of the forecasting series to the actual series suggests that the selected model has better prediction power and is appropriate to forecast CIT. This is confirmed by the calculated accuracy measures, i.e. RMSE of 2975.36, MAE of 2134.91, MAPE of 4.75 and a Theil U statistics of 0.03. The RMSE is smaller than that from the ARIMA method; hence, the ETS method of forecasting is better than that of ARIMA.

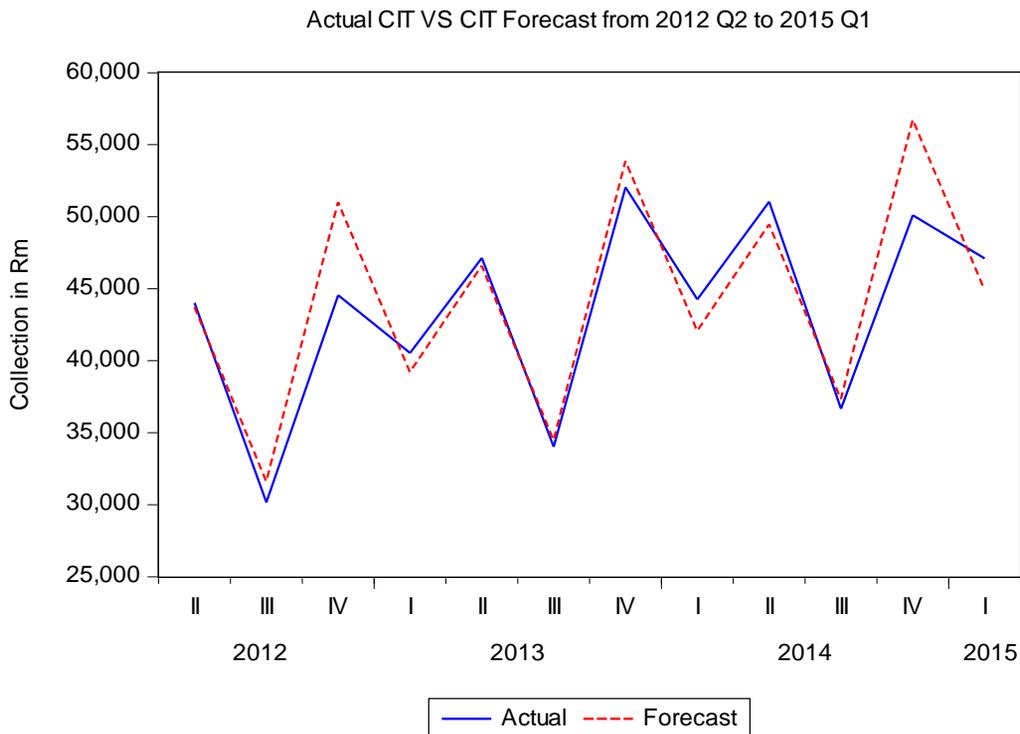


Figure 6: Forecasts for h=12 quarters ahead with an ETS (M, A, A) model

CIT BVAR Models

The CIT BVAR models are estimated using three priors, i.e. Minnesota prior, Normal-Wishart prior and Sims-Zha prior. Parameters in these priors are selected based on the combination of the ones suggested in the literature and a search over a range of possible hyper-parameters, seeking a combination that provides the best forecasting model with minimum RMSE. Doan (2007) proposes that the priors be selected symmetrically with an overall tightness of $\lambda_1 = 0.2$ and the relative weight $\lambda_2 = 0.1$ for small sized models. Caraianni (2010) estimates models with $\lambda_1 = 0.2$ and $\lambda_1 = 0.5$ with lag decay set to 1 and 2. In the studies of Korobolis (2009) as well as Kadiyala and Karlsson (1997), the relative weight was set to 0.005. Sims and Zha (2006) propose $\lambda_0 = 1$, $\lambda_1 = 0.2$, $\lambda_3 = 1$ and 1 for unit root and trend dummies.

In this study, the Minnesota prior parameters, $\mu_1, \lambda_1, \lambda_2$ and λ_3 are set to 0.5; 0.5; 0.6 and 0.1 respectively. The parameters for Normal-Wishart, μ_1 and λ_3 are set to 0.5 and 0.01 respectively. The Sims-Zha parameters, λ_0, γ_1 and λ_3 are set to 1, 0.9 and 0.9 respectively. The λ_1 represents the overall tightness parameter. It is the prior standard deviation of the coefficient of the first own lag and basically controls the prior standard deviations of all the other lag coefficients. This prior determines how all the coefficients are concentrated around their prior means. When the tighter prior is desired λ_1 must be

decreased. The λ_2 is the cross-variable weight tightness parameter; it represents the tightness of variable j in relation to variable i in equation i . Own lags generally account for most of the variation in a dependent variable, therefore the coefficients of cross lags are given smaller standard deviations than coefficients of own lags. λ_2 takes the value between 0 and 1. The λ_3 is a decay factor that controls the tightness on lag l relative to lag 1. As coefficients of higher order lags are more likely to approach zeros than those of lower order lags, prior standard deviations of coefficients decrease as lag length l increases. Table 5 shows the results of CIT BVAR models using the three priors.

Table 5: CIT BVAR models results with three priors (Standard error in brackets)

	Minnesota Prior			Normal Whishart Prior			Sims-Zha Prior		
	$DLCIT_t$	$DLGOS_t$	$DLRANDOL_t$	$DLCIT_t$	$DLGOS_t$	$DLRANDOL_t$	$DLCIT_t$	$DLGOS_t$	$DLRANDOL_t$
$DLCIT_{t-1}$	-0.631 (0.142)	-0.021 (0.019)	0.022 (0.045)	-0.778 (0.212)	-0.034 (0.157)	0.048 (0.163)	-0.670 (0.135)	-0.024 (0.021)	0.026 (0.046)
$DLCIT_{t-2}$	-0.283 (0.158)	-0.018 (0.020)	0.016 (0.048)	-0.435 (0.893)	-0.031 (0.661)	0.050 (0.689)	-0.295 (0.666)	-0.015 (0.102)	0.018 (0.227)
$DLCIT_{t-3}$	-0.342 (0.138)	-0.002 (0.018)	0.061 (0.043)	-0.452 (0.556)	-0.012 (0.412)	0.094 (0.429)	-0.336 (0.400)	0.000 (0.061)	0.060 (0.136)
$DLCIT_{t-4}$	0.163 (0.120)	-0.012 (0.016)	0.024 (0.038)	0.078 (0.241)	-0.024 (0.179)	0.048 (0.186)	0.139 (0.146)	-0.014 (0.022)	0.023 (0.050)
$DLGOS_{t-1}$	0.235 (0.659)	0.173 (0.101)	-0.082 (0.234)	0.364 (0.857)	0.219 (0.635)	-0.132 (0.661)	0.189 (0.636)	0.205 (0.098)	-0.095 (0.216)
$DLGOS_{t-2}$	1.716 (0.633)	-0.092 (0.099)	0.213 (0.226)	1.790 (0.558)	-0.094 (0.413)	0.195 (0.430)	1.846 (0.391)	-0.123 (0.060)	0.237 (0.133)
$DLGOS_{t-3}$	0.918 (0.677)	0.130 (0.105)	-0.060 (0.240)	1.181 (0.206)	0.161 (0.152)	-0.088 (0.159)	0.982 (0.122)	0.161 (0.019)	-0.088 (0.041)
$DLGOS_{t-4}$	-0.076 (0.688)	0.717 (0.106)	-0.115 (0.241)	0.165 (0.929)	0.697 (0.688)	-0.173 (0.716)	-0.086 (0.674)	0.629 (0.104)	-0.104 (0.229)
$DLRANDOL_{t-1}$	-0.027 (0.374)	0.112 (0.055)	0.380 (0.144)	0.028 (0.524)	0.145 (0.388)	0.363 (0.404)	-0.040 (0.346)	0.135 (0.053)	0.390 (0.118)
$DLRANDOL_{t-2}$	0.352 (0.372)	-0.015 (0.055)	-0.188 (0.145)	0.436 (0.177)	-0.028 (0.131)	-0.199 (0.137)	0.389 (0.104)	-0.023 (0.016)	-0.173 (0.036)
$DLRANDOL_{t-3}$	-0.173 (0.349)	-0.020 (0.052)	0.156 (0.136)	-0.160 (0.956)	-0.016 (0.708)	0.156 (0.737)	-0.206 (0.653)	-0.019 (0.100)	0.140 (0.222)
$DLRANDOL_{t-4}$	-0.136 (0.344)	-0.031 (0.051)	-0.173 (0.134)	-0.170 (0.515)	-0.040 (0.381)	-0.192 (0.397)	-0.156 (0.322)	-0.030 (0.050)	-0.129 (0.110)
RMSE	2690.400			2874.310			3416.080		

CIT BVAR Model Selection

We selected the best CIT BVAR model by comparing the RMSE of the out-sampling forecasts accuracy and the smallest is that of BVAR Minnesota prior with RMSE of 2690.4. Other RMSEs are 2874.31 for BVAR_{nw} and 3416.08 for BVAR_{sz}.

CIT BVAR Forecasts

We used the best model to generate CIT ($BVAR_{Minne}$) quarterly forecasts as presented in Figure 7.

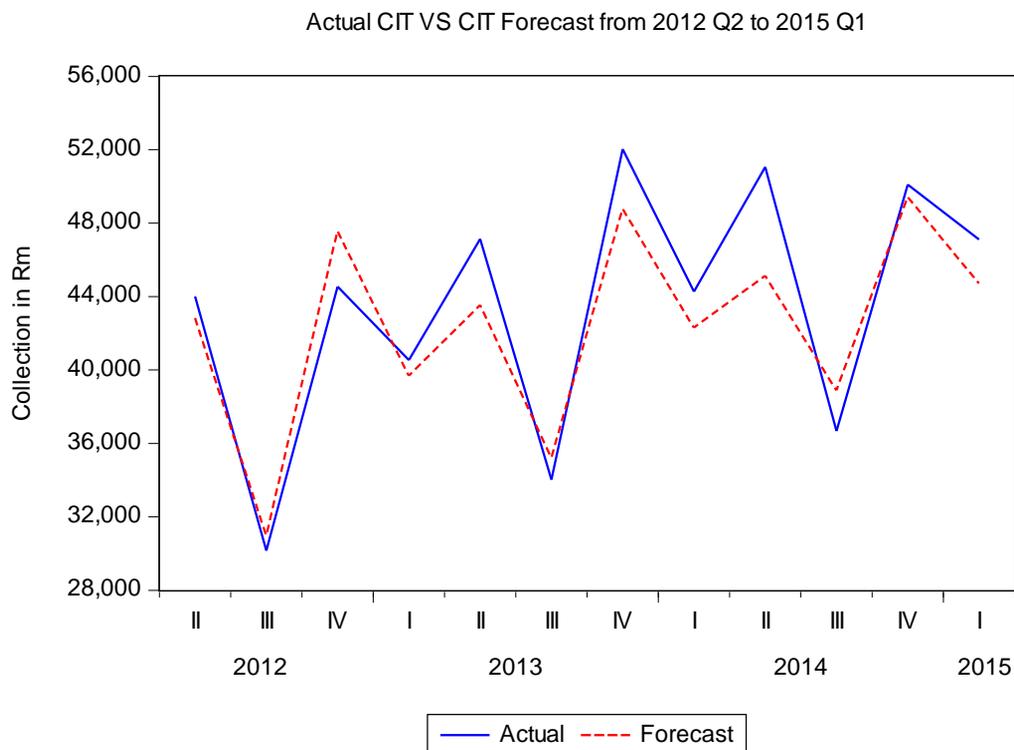


Figure 7: The forecasts for h=12 quarters ahead with a CIT BVAR model

In summary, for CIT, $BVAR_{minne}$ was superior to ARIMA (4,1,1) and ETS (M,A,A) in handling the CIT series. Therefore, the appropriate technique that may be used to forecast Corporate Income Tax is BVAR. Table 6 shows the RMSE of the three methods.

Table 6: RMSE for CIT models

<i>CIT Models</i>	<i>RMSE</i>
$BVAR_{minne}$	2690.40
ETS (M,A,A)	2975.36
ARIMA(4,1,1)	3847.81

Overall Results

The results for all the variables modelled by method and the best model selected in each category as evaluated by the RMSE criterion are presented in Table 7. The results show that the BVAR using Minnesota prior performs better than the ARIMA and ETS in all tax types under consideration, except for total tax revenue, which seems best fitted by ETS method.

Table 7: The RMSE of the forecasts for each tax type

<i>Model</i>	<i>CIT</i>	<i>PIT</i>	<i>VATP</i>	<i>TTR</i>
BVAR	BVAR _(minne) 2690.40	BVAR _(minne) 3201.30	BVAR _(minne) 645.69	BVAR _(minne) 5738.99
ETS	ETS(M,A,A) 2975.36	ETS(A,M,A) 3526.07	ETS(M,M,A) 1179.92	ETS(M,M _D ,A) 4976.50
ARIMA	ARIMA(4,1,1) 3847.81	ARIMA(4,1,0) 4509.41	ARIMA(3,1,0) 972.16	ARIMA(4,1,1) 9999.92

Discussion

The models' forecasting performance are evaluated by determining the root mean squared error for the out-of-sample forecasts between the second quarter of 2012 and the first quarter of 2015. Figure 8 shows the out-of-sample forecasts generated by three techniques (ARIMA, ETS and BVAR). Bayesian Vector Auto-regression with Minnesota priors performs well at individual tax level; it outperforms both ARIMA and ETS models. In the case of aggregated tax (TTR), BVAR fails to outperform both ARIMA and ETS models. The best model for TTR was *ETS (M, M_D, A)* which also outperforms ARIMA. BVAR is the second best, and it outperforms ARIMA. Table 7 gives the RMSE for all the methods and all the tax types. The reason associated with BVAR not performing against ETS model may be a likely misspecification of the TTR model. As stated earlier, TTR is dependent on some uncontrollable economic variables/data, which may have affected the BVAR forecasting.

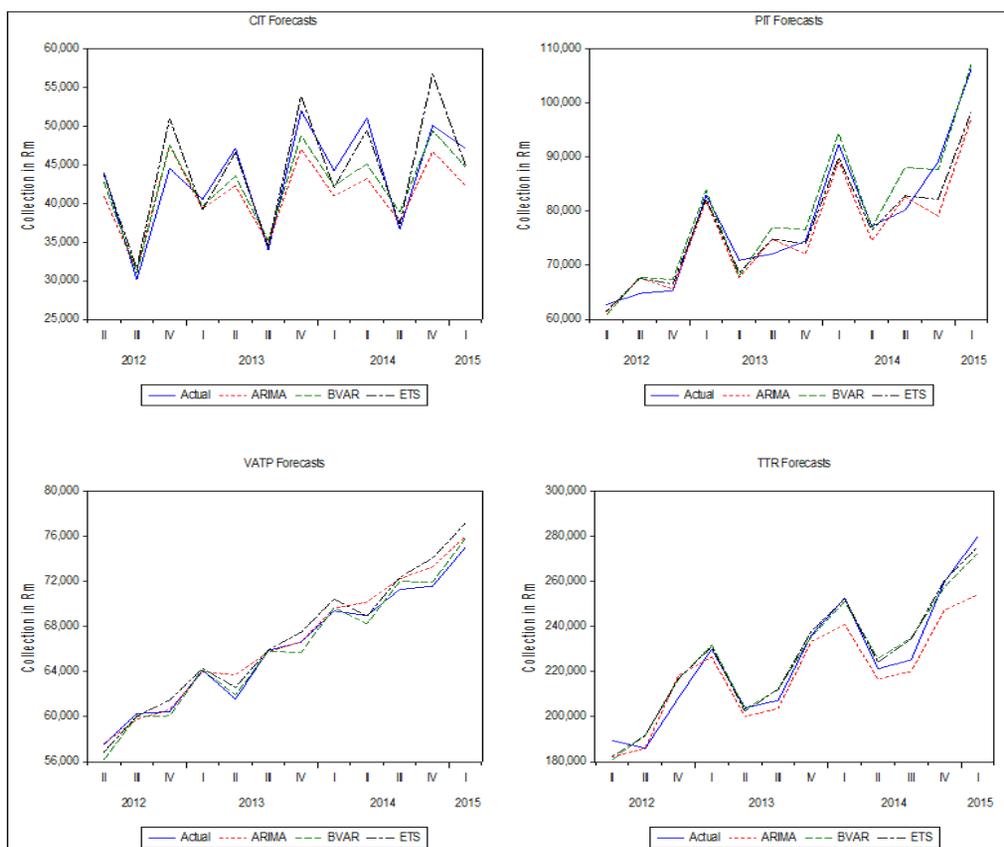


Figure 8: The charts showing forecasts generated by ARIMA, ETS and BVAR method

The results of this study are comparable to that of Krol (2010), though different hyper-parameters were used. In Krol (2010), the BVAR models perform better than VAR models for Sales Tax Revenue, Corporate Tax Revenue (CIT) and Total Tax Revenue except for Personal Income Tax Revenue (PIT). In our study, ETS outperforms BVAR for forecasting the TTR only. Shahnazarian et al. (2017) found that BVAR models are robust and produce reasonable conditional forecasts when compared with Direct Tax Revenue forecasts from a Mixed Data Sampling (MIDAS) equation, and typical naïve forecasts from Simple Integrated AR models with exogenous variables (ARIX). Our results corroborate this, as the BVAR method was found to be more robust than the ARIMA method in forecasting tax revenue for South Africa.

Conclusion

This paper models and forecasts the South African’s major tax revenues of CIT, PIT, VAT and Total Tax Revenue (TTR), using Bayesian Vector Auto-regression (BVAR),

Auto-regressive Moving Average (ARIMA) and State Space exponential smoothing (Error, Trend, Seasonal [ETS]) models, with quarterly data from 1998 to 2012Q1. The forecasts of the three models, based on Root mean square error (RMSE) forecasting accuracy measure, were from the out-of-sample period 2012Q2 to 2015Q1. Comparison of the performance (forecasting accuracy) of the BVAR approach with the ARIMA and ETS methods are made in order to recommend the best model for forecasting tax revenue in South Africa.

The results in this study confirm the accuracy of Bayesian Vector Auto-regression for predicting tax data. BVAR using Minnesota priors performs better than ARIMA and ETS in all taxes under consideration, except for total tax revenue (TTR). The ETS model best fits the TTR followed by BVAR_{minne}.

Notwithstanding the accuracy of the BVAR method in this study in predicting or forecasting tax revenues, the BVAR forecasts could be further improved by selecting more appropriate exogenous variables that explain various tax types and by including more economic variables. Heidari (2012) stated that in practice a VAR model with four variables and three lags is more common than a VAR model with four variables and one lag. Our study used three variables and four lags each in formulating our BVAR models for each tax type. More variables and more lags could be considered in future studies.

Gürkaynak, Kisacikoğlu and Rossi (2013) have alluded to the fact that there is no absolute best forecasting method. We, therefore, advise policymakers to incorporate the BVAR method amongst the existing methods employed in forecasting South African tax revenues. Studies on BVAR forecasting technique may also be extended to other smaller taxes to investigate whether it will fit these taxes accurately as it does for major taxes.

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